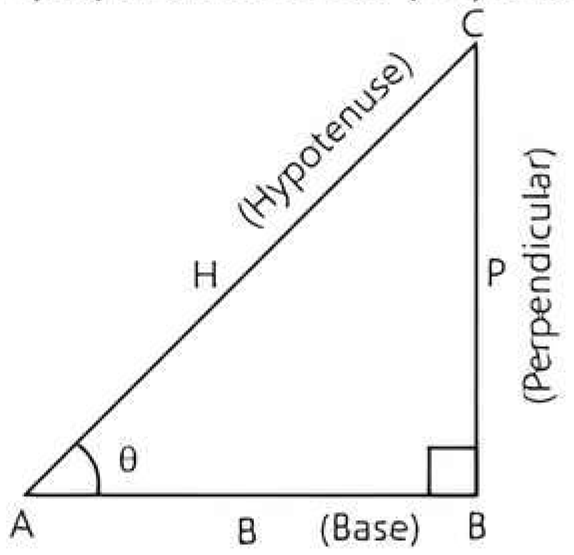
8 Introduction to Trigonometry

Fastrack Revision

- **Trigonometry:** It is the branch of Mathematics which deals with the measurement of angles and sides. 'Tri' means three, 'Gon' means sides and 'Metro' means measure.
- **Trigonometric Ratios:** Relations between different sides and angles of a right-angled triangle are called trigonometric ratios or T-ratios.
- In a right-angled ΔABC, $\angle B = 90°$, $\angle A = \theta$ (or $\angle C = 90° - \theta$) both are acute angles.
- Side opposite to right-angled $\angle B$ (90°) is known as hypotenuse (AC), side opposite to $\angle A$ (θ) is known as perpendicular (BC) and third side (AB) is known as base.



(i) $\sin\theta$ (sine of θ) $= \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{P}{H} = \frac{BC}{AC}$

(ii) $\cos\theta$ (cosine of θ) $= \frac{\text{Base}}{\text{Hypotenuse}} = \frac{B}{H} = \frac{AB}{AC}$

(iii) $\tan\theta$ (tangent of θ) $= \frac{\text{Perpendicular}}{\text{Base}} = \frac{P}{B} = \frac{BC}{AB}$

(iv) $\operatorname{cosec}\theta$ (cosecant of θ) $= \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{H}{P} = \frac{AC}{AB}$

(v) $\sec\theta$ (secant of θ) $= \frac{\text{Hypotenuse}}{\text{Base}} = \frac{H}{B} = \frac{AC}{AB}$

(vi) $\cot\theta$ (cotangent of θ) $= \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P} = \frac{AB}{BC}$

- **Relations between Trigonometric Ratios:**

(i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\sin\theta \cdot \operatorname{cosec}\theta = 1$

(ii) $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta \cdot \sec\theta = 1$

(iii) $\tan\theta = \frac{1}{\cot\theta}$ or $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta \cdot \cot\theta = 1$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be determined easily.
- **Pythagoras Theorem:** In a right-angled triangle, when two sides are given, then we find the third side using Pythagoras theorem.

$$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$$

$$H^2 = P^2 + B^2$$

or $\quad P^2 = H^2 - B^2 \quad$ or $\quad B^2 = H^2 - P^2$

- **Values of Trigonometric Ratios of Standard Angles**

T-Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Knowledge BOOSTER

1. *The value of $\sin\theta$ or $\cos\theta$ never exceeds 1, whereas the value of $\sec\theta$ or $\operatorname{cosec}\theta$ is always greater than or equal to 1.*
2. *The values of the trigonometric ratios of an angle depend only on the magnitude of the angle and not on the lengths of the sides of the triangle.*

- **Trigonometric Identities:** Trigonometric identities are equalities that involve T-ratios and are true for every value of the occurring variables where both sides of the equality are defined.

(i) $\sin^2\theta + \cos^2\theta = 1$ for $0° \le \theta \le 90°$

or $\cos^2\theta = 1 - \cos^2\theta$
or $\cos^2\theta = 1 - \sin^2\theta$ } Conversions

(ii) $1+\tan^2\theta = \sec^2\theta$ for $0° \le \theta < 90°$

$$\left.\begin{array}{ll} \text{or} & \tan^2\theta = \sec^2\theta - 1 \\ \text{or} & \sec^2\theta - \tan^2\theta = 1 \\ \text{or} & \tan^2\theta - \sec^2 = -1 \end{array}\right\} \text{Conversions}$$

(iii) $1+\cot^2\theta = \text{cosec}^2\theta$ for $0° < \theta \le 90°$

$$\left.\begin{array}{ll} \text{or} & \cot^2\theta = \text{cosec}^2\theta - 1 \\ \text{or} & \text{cosec}^2\theta - \cot^2\theta = 1 \\ \text{or} & \cot^2\theta - \text{cosec}^2\theta = -1 \end{array}\right\} \text{Conversions}$$

Practice Exercise

Multiple Choice Questions

Q 1. The hour-hand of a clock is 6 cm long. The angle swept by it between 7:20 am and 7:55 am is: *[CBSE 2023]*

a. $\left(\frac{35}{4}\right)^\circ$ b. $\left(\frac{35}{2}\right)^\circ$

c. $35°$ d. $70°$

Q 2. Given that $\sin\theta = \frac{a}{b}$, find $\cos\theta$. *[CBSE SQP 2023-24]*

a. $\frac{b}{\sqrt{b^2-a^2}}$ b. $\frac{b}{a}$

c. $\frac{\sqrt{b^2-a^2}}{b}$ d. $\frac{a}{\sqrt{b^2-a^2}}$

Q 3. In $\triangle ABC$ right-angled at B, if $\tan A = \sqrt{3}$, then $\cos A \cos C - \sin A \sin C =$ *[CBSE SQP 2021 Term-I]*

a. -1 b. 0

c. 1 d. $\sqrt{3}/2$

Q 4. If $\sin x + \text{cosec}\, x = 2$, then $\sin^{19}x + \text{cosec}^{20}x =$

a. 2^{19} b. 2^{20} c. 2 d. 2^{39}

Q 5. If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A =$

a. 196 b. 194 c. 192 d. 190

Q 6. If $\cot\theta = \frac{1}{\sqrt{3}}$, then the value of $\sec^2\theta + \text{cosec}^2\theta$ is: *[CBSE 2021 Term-I]*

a. 1 b. $\frac{40}{9}$

c. $\frac{38}{9}$ d. $5\frac{1}{3}$

Q 7. Given that, $\sec\theta = \sqrt{2}$, the value of $\frac{1+\tan\theta}{\sin\theta}$ is: *[CBSE 2021 Term-I]*

a. $2\sqrt{2}$ b. $\sqrt{2}$

c. $3\sqrt{2}$ d. 2

Q 8. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then $x =$ *[CBSE SQP 2022-23]*

a. $\cos 30°$ b. $\tan 30°$

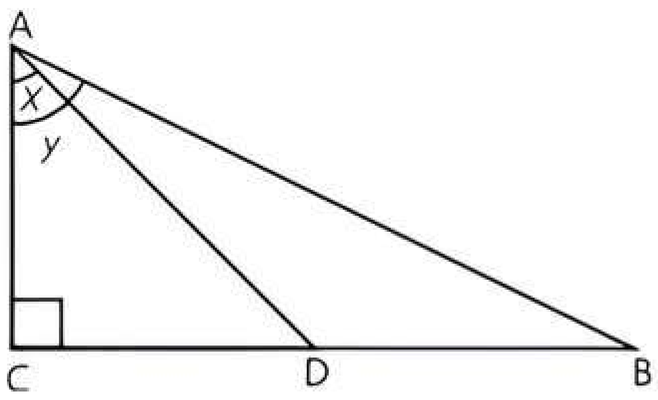
c. $\sin 30°$ d. $\cot 30°$

Q 9. If $\tan\alpha = \sqrt{3}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, $0 < \alpha, \beta < 90°$, then the value of $\cot(\alpha+\beta)$ is:

a. $\sqrt{3}$ b. 0

c. $\frac{1}{\sqrt{3}}$ d. 1

Q 10. In the given figure, if D is the mid-point of BC, then the value of $\frac{\cot y}{\cot x}$ is:



[CBSE SQP 2021 Term-I]

a. 2 b. 1/2 c. 1/3 d. 1/4

Q 11. $\sin 2A = 2\sin A$ is true when A is: *[NCERT EXERCISE]*

a. $0°$ b. $30°$ c. $45°$ d. $60°$

Q 12. If $5\tan\beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$ *[CBSE SQP 2022-23]*

a. $\frac{1}{3}$ b. $\frac{2}{5}$ c. $\frac{3}{5}$ d. 6

Q 13. If θ is an acute angle of a right angled triangle, then which of the following equation is not true? *[CBSE 2023]*

a. $\sin\theta\cot\theta = \cos\theta$ b. $\cos\theta\tan\theta = \sin\theta$

c. $\text{cosec}^2\theta - \cot^2\theta = 1$ d. $\tan^2\theta - \sec^2\theta = 1$

Q 14. $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$ in simplified form is: *[CBSE 2023]*

a. $\tan^2\theta$ b. $\sec^2\theta$ c. 1 d. -1

Q 15. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$ is equal to: *[CBSE 2023]*

a. -1 b. 1 c. 0 d. 2

Q 16. $(\sec A + \tan A)(1 - \sin A) =$ *[CBSE SQP 2023-24]*

a. sec A b. sin A c. cosec A d. cos A

Q 17. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$ is equal to:

a. 0 b. 1

c. -1 d. None of these

Q 18. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is: *[CBSE 2021 Term-I]*

a. 1 b. $\frac{1}{2}$

c. $\frac{\sqrt{2}}{2}$ d. $\sqrt{2}$

Q 19. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$ [CBSE 2020, CBSE SQP 2022-23]

a. 1 b. 2 c. 3 d. 4

Q 20. If $a\cot\theta + b\,\text{cosec}\,\theta = p$ and $b\cot\theta + a\,\text{cosec}\,\theta = q$, then $p^2 - q^2 =$ [CBSE 2021 Term-I]

a. $a^2 - b^2$ b. $b^2 - a^2$
c. $a^2 + b^2$ d. $b - a$

Q 21. If the angles of $\triangle ABC$ are in ratio 1 : 1 : 2, respectively (the largest angle being angle C), then the value of $\frac{\sec A}{\text{cosec}\, B} - \frac{\tan A}{\cot B}$ is: [CBSE SQP 2021 Term-I]

a. 0 b. 1/2
c. 1 d. $\sqrt{3}/2$

Q 22. Evaluate $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$.

a. $2\sin\theta$ b. $2\cos\theta$
c. $2\,\text{cosec}\,\theta$ d. $2\sec\theta$

Q 23. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is: [NCERT EXEMPLAR; CBSE 2020]

a. 1 b. $\frac{1}{2}$ c. 2 d. 3

Q 24. If $2\sin^2\beta - \cos^2\beta = 2$, then β is: [CBSE SQP 2021 Term-I]

a. 0° b. 90° c. 45° d. 30°

Q 25. If $\tan\alpha + \cot\alpha = 2$, then $\tan^{20}\alpha + \cot^{20}\alpha =$ [CBSE SQP 2021 Term-I]

a. 0 b. 2 c. 20 d. 2^{20}

Q 26. The value of θ in $5\sin^2\theta - \cos^2\theta = 2$ is:

a. 30° b. 45° c. 60° d. 90°

Q 27. Which of the following is true for all values of θ $(0° \le \theta \le 90°)$? [CBSE 2023]

a. $\cos^2\theta - \sin^2\theta = 1$ b. $\text{cosec}^2\theta - \sec^2\theta = 1$
c. $\sec^2\theta - \tan^2\theta = 1$ d. $\cot^2\theta - \tan^2\theta = 1$

Q 28. If $\text{cosec}\,\theta - \cot\theta = \frac{1}{3}$, the value of $(\text{cosec}\,\theta + \cot\theta)$ is: [CBSE 2021 Term-I]

a. 1 b. 2 c. 3 d. 4

Q 29. If $\sec\theta + \tan\theta = p$, then $\tan\theta$ is: [CBSE 2021 Term-I]

a. $\frac{p^2+1}{2p}$ b. $\frac{p^2-1}{2p}$ c. $\frac{p^2-1}{p^2+1}$ d. $\frac{p^2+1}{p^2-1}$

Q 30. If $1 + \sin^2\alpha = 3\sin\alpha\cos\alpha$, then the values of $\cot\alpha$ are: [CBSE SQP 2021 Term-I]

a. – 1, 1 b. 0, 1
c. 1, 2 d. – 1, –1

Assertion & Reason Type Questions

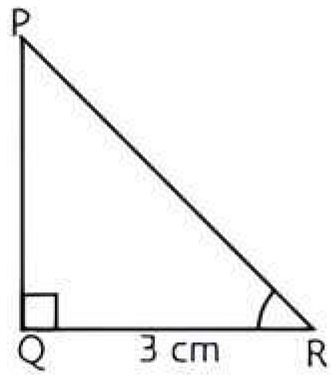
Directions (Q. Nos. 31-36): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 31. Assertion (A): The value of each of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

Reason (R): In right angled $\triangle ABC$, $\angle B = 90°$ and $\angle A = \theta$, $\sin\theta = \frac{BC}{AC} < 1$ and $\cos\theta = \frac{AB}{AC} < 1$ as hypotenuse is the longest side.

Q 32. Assertion (A): In $\triangle PQR$ right-angled at Q, QR = 3 cm and PR – PQ = 1 cm. The value of $\sin^2 R + \text{cosec}\, R$ is $\frac{189}{100}$.



Reason (R): $\sin^2 A = (\sin A)^2$ and $\text{cosec}\, A = (\sec A)^{-1}$.

Q 33. Assertion (A): ABCD is a rectangle such that $\angle CAB = 60°$ and AC = a units. The area of rectangle ABCD is $\frac{\sqrt{3}}{2}a^2$.

Reason (R): The value of sin 60° is $\frac{\sqrt{3}}{2}$ and cos 60° is $\frac{1}{2}$.

Q 34. Assertion (A): If $\sin\theta = \frac{1}{2}$ and θ is acute angle, then $(3\cos\theta - 4\cos^3\theta)$ is equal to 0.

Reason (R): As $\sin\theta = \frac{1}{2}$ and θ is acute, so θ must be 60°.

Q 35. Assertion (A): In a right-angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

Reason (R): $(\text{Greatest side})^2 = (\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$.

Q 36. Assertion (A): For $0° < \theta \le 90°$, $\text{cosec}\,\theta - \cot\theta$ and $\text{cosec}\,\theta + \cot\theta$ are reciprocal of each other.

Reason (R): $\text{cosec}^2\theta - \cot^2\theta = 1$. (CBSE 2023)

Fill in the Blanks Type Questions

Q 37. The value of $\sin\theta$ or $\cos\theta$ never exceeds ....

Q 38. In $\triangle ABC$ right-angled at point B, if $\tan A = \frac{1}{\sqrt{3}}$, then the value of $\sin A\cos C + \cos A\sin C$ is ....

Q 39. The minimum value of sec θ is .... *[NCERT EXERCISE]*

Q 40. If $\sin\theta - \cos\theta = \frac{1}{4}$, then $\sin\theta \cdot \cos\theta =$ ....

True/False Type Questions

Q 41. The values of the trigonometric ratios of an angle depend only on the magnitude of the angle and not on the length of the sides of the triangle.

Q 42. The value of tan θ increases from 0 to ∞ when θ increases from 0° to 90°.

Q 43. If $7\sin^2 A + 3\cos^2 A = 4$, then $\tan A = \frac{1}{\sqrt{3}}$.

Q 44. One of the trigonometric identity is $\sec^2\theta - \tan^2\phi = 1$.

Q 45. If $\sin\theta = \frac{1}{2}$, then the value of $2\cot^2\theta + 2$ is 8.

Solutions

1. (b) We know that,

Angle subtended by hour hand in 12 hours = 360°

$\therefore$ Angle subtended by hour hand in 1 hour $= \frac{360^\circ}{12} = 30^\circ$

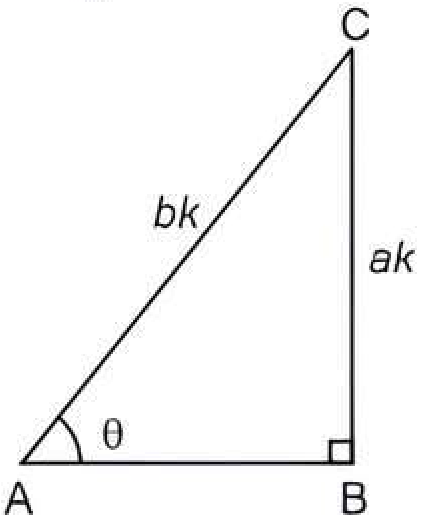
$\because$ Time from 7:20 am to 7:55 am = 35 minutes $= \frac{35}{60}$ hour

$\therefore$ Angle subtended by minute hand in $\frac{35}{60}$ hour

$$= \left(30 \times \frac{35}{60}\right)^\circ = \left(\frac{35}{2}\right)^\circ$$

2. (c) Given, $\sin\theta = \frac{a}{b} = \frac{L}{K}$

Construct a right triangle ABC in which ∠B = 90° and perpendicular BC = ak, hypotenuse AC = bk, where k is a positive integer.



In right angled △ABC,

$$AC^2 = AB^2 + BC^2 \text{ (by Pythagoras theorem)}$$

$$\Rightarrow AB^2 = AC^2 - BC^2 = (bk)^2 - (ak)^2 = (b^2 - a^2)k^2$$

$$\therefore AB = k\sqrt{b^2 - a^2}$$

$$\therefore \cos\theta = \frac{A}{K} = \frac{AB}{AC} = \frac{k\sqrt{b^2-a^2}}{bk} = \frac{\sqrt{b^2-a^2}}{b}$$

3. (b) We have, $\tan A = \sqrt{3}$

$$\Rightarrow \tan A = \tan 60^\circ$$

$$\Rightarrow A = 60^\circ$$

In right-angled △ABC,

$$A + B + C = 180^\circ$$

$$\Rightarrow 60^\circ + 90^\circ + C = 180^\circ \quad (\because B = 90^\circ)$$

$$C = 180^\circ - 150^\circ = 30^\circ$$

$\therefore \cos A \cdot \cos C - \sin A \cdot \sin C$

$$= \cos 60^\circ \cdot \cos 30^\circ - \sin 60^\circ \cdot \sin 30^\circ = \frac{1}{2}\times\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

4. (c) Given, $\sin x + \operatorname{cosec} x = 2$

$$\Rightarrow \sin x + \frac{1}{\sin x} = 2 \Rightarrow \sin^2 x + 1 = 2\sin x$$

$$\Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$$

$$\Rightarrow \operatorname{cosec} x = 1$$

$$\therefore \sin^{19} x + \operatorname{cosec}^{20} x = 1 + 1 = 2$$

5. (b) Given, $\tan A + \cot A = 4$

Squaring both sides, we get

$$(\tan A + \cot A)^2 = 4^2$$

$$\Rightarrow \tan^2 A + \cot^2 A + 2 = 16$$

$$\Rightarrow \tan^2 A + \cot^2 A = 14$$

Again squaring both sides,

$$(\tan^2 A + \cot^2 A)^2 = (14)^2$$

$$\Rightarrow \tan^4 A + \cot^4 A + 2 = 196$$

$$\Rightarrow \tan^4 A + \cot^4 A = 194$$

6. (d) Given, $\cot\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \cot\theta = \cot 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

$$\therefore \sec^2\theta + \operatorname{cosec}^2\theta = \sec^2 60^\circ + \operatorname{cosec}^2 60^\circ = (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 = 4 + \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$$

7. (a) Given, $\sec\theta = \sqrt{2}$

$$\Rightarrow \sec\theta = \sec 45^\circ$$

$$\Rightarrow \theta = 45^\circ$$

$$\therefore \frac{1+\tan\theta}{\sin\theta} = \frac{1+\tan 45^\circ}{\sin 45^\circ}$$

$$= \frac{1+1}{\frac{1}{\sqrt{2}}} = \frac{2\times\sqrt{2}}{1}$$

$$= 2\sqrt{2}$$

8. (b) Given,

$x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$

$$\therefore \quad x\times\sqrt{3}\times\frac{1}{2} = \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}}$$

$$= \tan 30^\circ$$

9. (b) We have, $\tan\alpha = \sqrt{3}$

$$\Rightarrow \quad \alpha = 60^\circ > 0^\circ \qquad (\because \tan 60^\circ = \sqrt{3})$$

and $\quad \tan\beta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \beta = 30^\circ < 90^\circ \qquad \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$

$$\therefore \quad \alpha+\beta = 60^\circ + 30^\circ = 90^\circ$$

$$\therefore \quad \cot(\alpha+\beta) = \cot 90^\circ = 0$$

10. (b) Given, D is the mid-point of BC.

$$\therefore \quad CD = BD = \frac{BC}{2} \qquad \text{...(1)}$$

Now in right-angled ΔACD,

$$\cot x = \frac{AC}{CD} = \frac{AC}{(BC/2)} = \frac{2AC}{BC} \qquad \text{[from eq. (1)] ...(2)}$$

In right-angled ΔACB,

$$\cot y = \frac{AC}{BC} \qquad \text{...(3)}$$

Dividing eq. (3) by eq. (2), we get

$$\frac{\cot y}{\cot x} = \frac{AC}{BC}\times\frac{BC}{2AC} = \frac{1}{2}$$

11. (a) From option (a),

$$LHS = \sin 2A = \sin 2\times 0^\circ = \sin 0^\circ = 0$$

and $\quad RHS = 2\sin A = 2\sin 0^\circ$

$$= 2\times 0 = 0$$

$$\because \quad LHS = RHS$$

$$\therefore \quad A = 0^\circ$$

12. (a) Given, $5\tan\beta = 4 \Rightarrow \tan\beta = \frac{4}{5}$

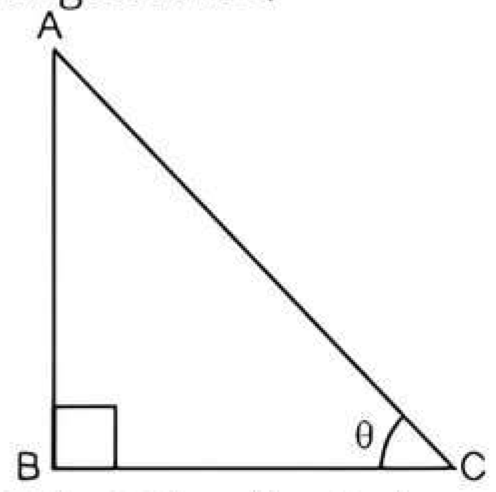
$$\therefore \quad \frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} = \frac{5\tan\beta - 2}{5\tan\beta + 2}$$

$$= \frac{5\times\frac{4}{5}-2}{5\times\frac{4}{5}+2} = \frac{4-2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

13. (d) Given, $\quad \theta$ = acute angle

Let $\quad \angle B = 90^\circ$

$\therefore$ In right angled ΔABC,



$$AC^2 = AB^2 + BC^2 \qquad \text{(by Pythagoras theorem) ...(1)}$$

Option (a): L.H.S. $= \sin\theta\cdot\cot\theta$

$$= \frac{AB}{AC}\times\frac{BC}{AB} = \frac{BC}{AC} = \cos\theta = \text{R.H.S.}$$

Option (b): L.H.S. $= \cos\theta\cdot\tan\theta$

$$= \frac{BC}{AC}\times\frac{AB}{BC} = \frac{AB}{AC} = \sin\theta = \text{R.H.S.}$$

Option (c): L.H.S. $= \operatorname{cosec}^2\theta - \cot^2\theta$

$$= \left(\frac{AC}{AB}\right)^2 - \left(\frac{BC}{AB}\right)^2 = \frac{AC^2}{AB^2} - \frac{BC^2}{AB^2}$$

$$= \frac{AC^2 - BC^2}{AB^2} = \frac{AB^2}{AB^2} = 1 \quad \text{[from eq. (1)]}$$

$$= \text{R.H.S.}$$

Option (d): L.H.S. $= \tan^2\theta - \sec^2\theta$

$$= \left(\frac{AB}{BC}\right)^2 - \left(\frac{AC}{BC}\right)^2 = \frac{AB^2}{BC^2} - \frac{AC^2}{BC^2}$$

$$= \frac{AB^2 - AC^2}{BC^2} = \frac{-BC^2}{BC^2} = -1 \qquad \text{[from eq. (1)]}$$

$$\neq \text{R.H.S.}$$

14. (d) $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta - 1}{\sin^2\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$$= \frac{-\sin^2\theta}{\sin^2\theta} = -1$$

15. (b) $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) \left(\begin{array}{l}\because 1+\tan^2\theta = \sec^2\theta \text{ and} \\ 1+\cot^2\theta = \operatorname{cosec}^2\theta\end{array}\right)$

$$= \tan^2\theta\cdot\cot^2\theta$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\times\frac{\cos^2\theta}{\sin^2\theta} = 1.$$

16. (d) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1+\sin A)}{\cos A}\times(1-\sin A) = \frac{1-\sin^2 A}{\cos A}$$

$$(\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

17. (c) We have, $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$$= 2(\sin^6\theta + \cos^6\theta - \sin^4\theta - \cos^4\theta) - (\sin^4\theta + \cos^4\theta)$$

$= 2\sin^4\theta(\sin^2\theta - 1) + 2\cos^4\theta(\cos^2\theta - 1) - (\sin^4\theta + \cos^4\theta)$

$= -2\sin^4\theta\cos^2\theta - 2\cos^4\theta\sin^2\theta - (\sin^4\theta + \cos^4\theta)$

$(\because \sin^2\theta - 1 = -\cos^2\theta, \cos^2\theta - 1 = -\sin^2\theta)$

$= -2\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) - (\sin^4\theta + \cos^4\theta)$

$= -(2\sin^2\theta\cos^2\theta + \sin^4\theta + \cos^4\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= -(\sin^2\theta + \cos^2\theta)^2 = -(1)^2 = -1$

18. (c) Given, $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 - 2\tan\theta = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0 \quad \Rightarrow \tan\theta - 1 = 0$

$\Rightarrow \tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$

$\therefore \sin^3\theta + \cos^3\theta = \sin^3 45° + \cos^3 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3$

$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

19. (b) Given, $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides,

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 2$

$\Rightarrow 2\sin\theta\cos\theta = 1$

$\Rightarrow \sin\theta\cos\theta = \frac{1}{2}$ ...(1)

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{1/2}$ [from eq. (1)]

$= 2$

20. (b) We have, $a\cot\theta + b\,\text{cosec}\,\theta = p$ ...(1)

and $b\cot\theta + a\,\text{cosec}\,\theta = q$ ...(2)

Squaring and then subtracting eq. (2) from eq. (1), we get

$p^2 - q^2 = a^2\cot^2\theta + b^2\text{cosec}^2\theta + 2ab\cot\theta\,\text{cosec}\,\theta - b^2\cot^2\theta - a^2\text{cosec}^2\theta - 2ab\cot\theta\,\text{cosec}\,\theta$

$= a^2(\cot^2\theta - \text{cosec}^2\theta) + b^2(\text{cosec}^2\theta - \cot^2\theta)$

$= -a^2 + b^2 \quad (\because \text{cosec}^2\theta - \cot^2\theta = 1)$

$= b^2 - a^2$

21. (a) Given angles of ΔABC are in the ratio 1 : 1 : 2 respectively.

Let $A = x$, $B = x$ and $C = 2x$

TiP

Sum of internal angles in a triangle is 180°.

In ΔABC,

$A + B + C = 180°$

$\Rightarrow x + x + 2x = 180°$

$\Rightarrow 4x = 180° \Rightarrow x = 45°$

$A = x = 45°$,

$B = x = 45°$,

and $C = 2x = 2 \times 45° = 90°$

Now, $\frac{\sec A}{\text{cosec}\,B} - \frac{\tan A}{\cot B} = \frac{\sec 45°}{\text{cosec}\,45°} - \frac{\tan 45°}{\cot 45°}$

$= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{1} = 1 - 1 = 0$

22. (c) We have, $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

$= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} \quad (\because \sec^2\theta - 1 = \tan^2\theta)$

$= \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta$

23. (a) Given, $\sin A + \sin^2 A = 1$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$\Rightarrow \sin A = 1 - \sin^2 A = \cos^2 A$

$(\because \cos^2 A + \sin^2 A = 1)$

Squaring both sides, we get

$\sin^2 A = \cos^4 A$

$1 - \cos^2 A = \cos^4 A \Rightarrow \cos^2 A + \cos^4 A = 1$

TiP

Sometimes students forget the identity used and hence commit error.

24. (b) Given, $2\sin^2\beta - \cos^2\beta = 2$

$\Rightarrow -\cos^2\beta = 2(1 - \sin^2\beta)$

$\Rightarrow -\cos^2\beta = 2\cos^2\beta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow 3\cos^2\beta = 0$

$\Rightarrow \cos^2\beta = 0$

$\Rightarrow \cos\beta = 0 = \cos 90°$

$\therefore \beta = 90°$

25. (b) Given, $\tan\alpha + \cot\alpha = 2$

$\Rightarrow \tan\alpha + \frac{1}{\tan\alpha} = 2$

$\Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$

$\Rightarrow \tan^2\alpha + 1 - 2\tan\alpha = 0$

$\Rightarrow (\tan\alpha - 1)^2 = 0$

$\Rightarrow \tan\alpha - 1 = 0$

$\Rightarrow \tan\alpha = 1 = \tan 45°$

$\therefore \alpha = 45°$

Now, $\tan^{20}\alpha + \cot^{20}\alpha = (\tan 45°)^{20} + (\cot 45°)^{20}$

$= (1)^{20} + (1)^{20}$

$= 1 + 1 = 2$

26. (b) $5\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow 5\sin^2\theta - (1 - \sin^2\theta) = 2$

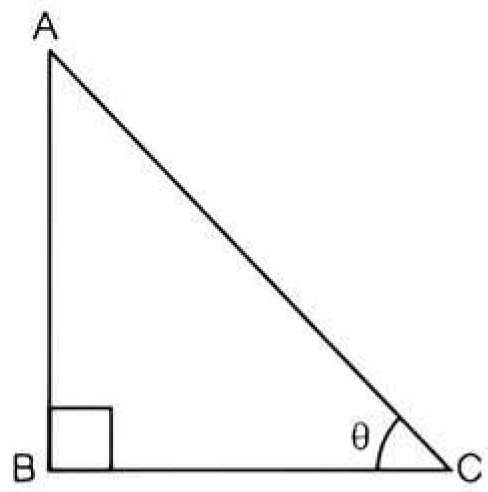
$\Rightarrow 6\sin^2\theta = 3 \Rightarrow \sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} = \sin 45° \quad \Rightarrow \theta = 45°$

27. (c) Let $\angle B = 90°$ and $0° \le \theta \le 90°$

$\therefore$ In right ΔABC,

$AC^2 = AB^2 + BC^2$ (by Pythagoras theorem) ...(1)



Option (a): L.H.S. $= \cos^2\theta - \sin^2\theta$

$$= \left(\frac{BC}{AC}\right)^2 - \left(\frac{AB}{AC}\right)^2 = \frac{BC^2 - AB^2}{AC^2}$$

$\neq$ R.H.S.

Option (b): L.H.S. $= \text{cosec}^2\theta - \sin^2\theta$

$$= \left(\frac{AC}{AB}\right)^2 - \left(\frac{AC}{BC}\right)^2$$

$$= \frac{AC^2\,(BC^2 - AB^2)}{AB^2 \cdot BC^2} \neq \text{R.H.S.}$$

Option (c): L.H.S. $= \sec^2\theta - \tan^2\theta$

$$= \left(\frac{AC}{BC}\right)^2 - \left(\frac{AB}{BC}\right)^2 = \frac{AC^2 - AB^2}{BC^2}$$

$$= \frac{BC^2}{BC^2} \quad \text{[from eq. (1)]}$$

$= 1 =$ R.H.S.

Option (d): L.H.S. $= \cot^2\theta - \tan^2\theta$

$$= \left(\frac{BC}{AB}\right)^2 - \left(\frac{AB}{BC}\right)^2 = \frac{BC^4 - AB^4}{AB^2 \cdot BC^2}$$

$=$ R.H.S.

28. (c) Given, $\text{cosec}\,\theta - \cot\theta = \frac{1}{3}$

Also, $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$

$\Rightarrow \quad (\text{cosec}\,\theta + \cot\theta) \times \frac{1}{3} = 1$

$\Rightarrow \quad \text{cosec}\,\theta + \cot\theta = 3$

29. (b) Given, $\sec\theta + \tan\theta = p$ .... (1)

$\because \quad \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$\Rightarrow \quad (\sec\theta - \tan\theta)\,p = 1$

$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{p}$ .... (2)

Subtracting eq. (2) from eq. (1), we get

$$2\tan\theta = p - \frac{1}{p}$$

$$\Rightarrow \quad \tan\theta = \frac{p^2 - 1}{2p}$$

30. (c) Given, $1 + \sin^2\alpha = 3\sin\alpha \cdot \cos\alpha$

$\Rightarrow \quad 1 + 1 - \cos^2\alpha = 3\sin\alpha \cdot \cos\alpha$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad 2 - \cos^2\alpha = 3\sin\alpha \cdot \cos\alpha$

$$\Rightarrow \quad \frac{2 - \cos^2\alpha}{\sin^2\alpha} = \frac{3\sin\alpha \cdot \cos\alpha}{\sin^2\alpha}$$

(dividing both sides by $\sin^2\alpha$)

$\Rightarrow \quad 2\,\text{cosec}^2\alpha - \cot^2\alpha = 3\cot\alpha$

$\Rightarrow \quad 2(1 + \cot^2\alpha) - \cot^2\alpha = 3\cot\alpha$

$\Rightarrow \quad \cot^2\alpha - 3\cot\alpha + 2 = 0$

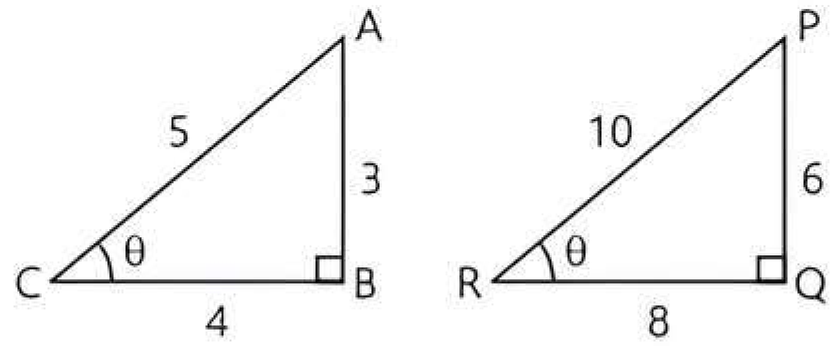
$\Rightarrow \quad \cot^2\alpha - 2\cot\alpha - \cot\alpha + 2 = 0$

$\Rightarrow \quad \cot\alpha(\cot\alpha - 2) - 1(\cot\alpha - 2) = 0$

$\Rightarrow \quad (\cot\alpha - 2)(\cot\alpha - 1) = 0$

$\Rightarrow \quad \cot\alpha = 1, 2$

31. (b) **Assertion (A):** Suppose in ΔABC and in ΔPQR



$$\sin\theta = \frac{AB}{AC} = \frac{3}{5} \text{ and } \sin\theta = \frac{PQ}{PR} = \frac{6}{10}$$

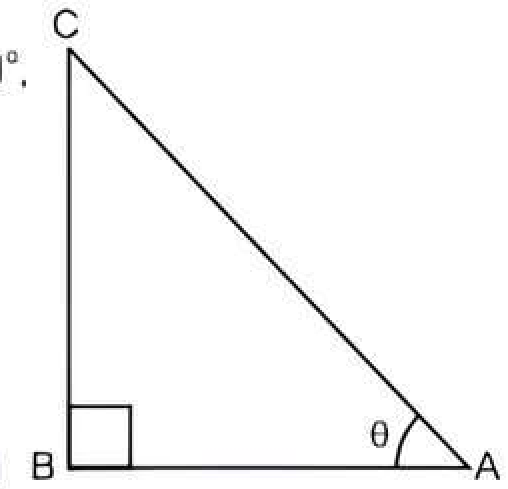
$$\Rightarrow \quad \sin\theta = \frac{3}{5} \text{ and } \sin\theta = \frac{3}{5}$$

Similarly, this will also holds for other trigonometric ratios.

So, trigonometric ratio does not depend on the size of the triangle.

So, Assertion (A) is true.

Reason (R): Given, $\angle B = 90°$, $\angle A = \theta$ and



$$\sin\theta = \frac{BC}{AC} < 1$$

$$\text{and } \cos\theta = \frac{AB}{AC} < 1$$

$\therefore \quad \sin^2\theta + \cos^2\theta < 1$

$$\Rightarrow \quad \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2 < 1$$

$$\Rightarrow \quad \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2} < 1$$

$\Rightarrow \quad AB^2 + BC^2 < AC^2$

So, in right ΔABC hypotenuse is the longest side.

$\therefore$ Reason (R) is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

32. (c) **Assertion (A):** Since, ΔPQR is right-angled triangle.

From Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow \quad QR^2 = PR^2 - PQ^2$

$\Rightarrow \quad (3)^2 = PR^2 - PQ^2 \; (\because QR = 3\text{ cm})$

$\Rightarrow \quad PR^2 - PQ^2 = 9$

$\Rightarrow \quad (PR + PQ)(PR - PQ) = 9 \; (\because a^2 - b^2 = (a+b)(a-b))$

$\Rightarrow \quad (PR + PQ)\,1 = 9 \quad (\because PR - PQ = 1\text{ cm})$

$\Rightarrow \quad PR + PQ = 9$

On solving PR + PQ = 9 and PR – PQ = 1, we get PR = 5 and PQ = 4

$$\therefore \quad \sin R = \frac{PQ}{PR} = \frac{4}{5}$$

So, $\sin^2 R + \text{cosec } R = \sin^2 R + \frac{1}{\sin R}$

$$= \left(\frac{4}{5}\right)^2 + \frac{1}{4/5} = \frac{16}{25} + \frac{5}{4} = \frac{64+125}{25 \times 4} = \frac{189}{100}$$

So, Assertion (A) is true.

Reason (R): Also, cosec A = $(\sin A)^{-1}$

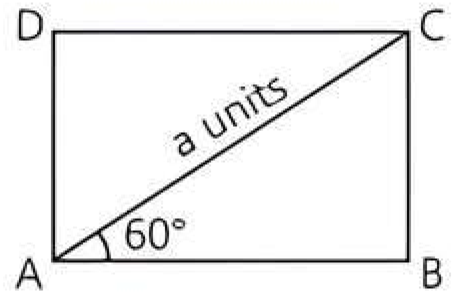
i.e., $\text{cosec } A \neq (\sec A)^{-1}$

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

33. (d) **Assertion (A):** In ΔABC, AC = a units, $\angle A = 60°$

$$\therefore \quad \sin 60° = \frac{BC}{AC} = \frac{BC}{a}$$



$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BC}{a} \quad \Rightarrow BC = \frac{a\sqrt{3}}{2}$$

Also, $\cos 60° = \frac{AB}{AC} \quad \Rightarrow \frac{1}{2} = \frac{AB}{a}$

$\Rightarrow \quad AB = a/2$

$\therefore$ Area of rectangle ABCD = AB × BC

$$= \frac{a}{2} \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4} a^2$$

So, Assertion (A) is false.

Reason (R): It is true to say that the value of sin 60° is $\frac{\sqrt{3}}{2}$ and cos 60° is $\frac{1}{2}$.

Hence, Assertion (A) is false but Reason (R) is true.

34. (c) **Assertion (A):** We have, $\sin\theta = \frac{1}{2}$

$$\Rightarrow \quad \theta = 30° \quad \left(\because \sin 30° = \frac{1}{2}\right)$$

$$\therefore \quad 3\cos\theta - 4\cos^3\theta = 3\cos 30° - 4\cos^3 30°$$

$$= \frac{3\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

So, Assertion (A) is true.

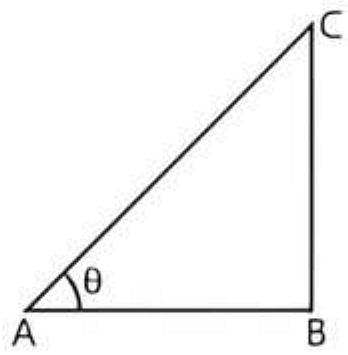
Reason (R): It is false to say that at $\theta = 60°$, $\sin\theta = \frac{1}{2}$.

This will be correct at $\theta = 30°$.

Hence, Assertion (A) is true but Reason (R) is false.

35. (a) **Assertion (A):** Given,

$$\tan\theta = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$



Let BC = $3k$ and AB = $4k$

In right-angled ΔABC, by Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{(4k)^2 + (3k)^2} = \sqrt{16k^2 + 9k^2}$$

$$= \sqrt{25k^2} = 5k$$

It is true to say that greatest side of a triangle is hypotenuse.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

36. (a) **Assertion (A):** For $0° < \theta \le 90°$, $\text{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow \quad (\text{cosec }\theta - \cot\theta)(\text{cosec }\theta + \cot\theta) = 1$$

$$[\because a^2 - b^2 = (a-b)(a+b)]$$

$$\Rightarrow \quad \text{cosec }\theta - \cot\theta = \frac{1}{\text{cosec }\theta + \cot\theta}$$

Or

$$\text{cosec }\theta + \cot\theta = \frac{1}{\text{cosec }\theta - \cot\theta}$$

$\therefore$ $(\text{cosec }\theta - \cot\theta)$ and $(\text{cosec }\theta + \cot\theta)$ are reciprocal of each other.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

37. one

38. Given, $\tan A = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan A = \tan 30°$

$\Rightarrow \quad A = 30°$

In right-angled ΔABC,

$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad 30° + 90° + \angle C = 180°$$

$$\Rightarrow \quad \angle C = 60°$$

$$\because \sin A \cos C + \cos A \sin C = \sin 30° \cos 60° + \cos 30° \sin 60°$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4}$$

$$= 1$$

39. The minimum value of sec θ is one.

40. Given, $\sin\theta - \cos\theta = \frac{1}{4}$

Squaring on both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{16}$$

$$\Rightarrow \quad 1 - 2\sin\theta\cos\theta = \frac{1}{16}$$

$$\Rightarrow \quad 2\sin\theta\cos\theta = 1 - \frac{1}{16}$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2}\left(\frac{15}{16}\right) = \frac{15}{32}$$

41. True

42. True

43. Given, $7\sin^2 A + 3\cos^2 A = 4$

$\Rightarrow 7(1 - \cos^2 A) + 3\cos^2 A = 4$

$\Rightarrow 7 - 4\cos^2 A = 4$

$\Rightarrow 4\cos^2 A = 3$

$\Rightarrow \cos^2 A = \frac{3}{4}$

$\Rightarrow \cos A = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos A = \cos 30°$

$\Rightarrow A = 30°$

$\therefore \tan A = \tan 30°$

$\Rightarrow \tan A = \frac{1}{\sqrt{3}}$

Hence, given statement is true.

44. In the given equation, both trigonometric ratios have different angles, So, the given equation will not have trigonometric identity.

Hence, given statement is false.

45. Given, $\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

$\therefore 2\cot^2\theta + 2 = 2\cot^2 30° + 2$

$= 2(\sqrt{3})^2 + 2$

$= 2 \times 3 + 2$

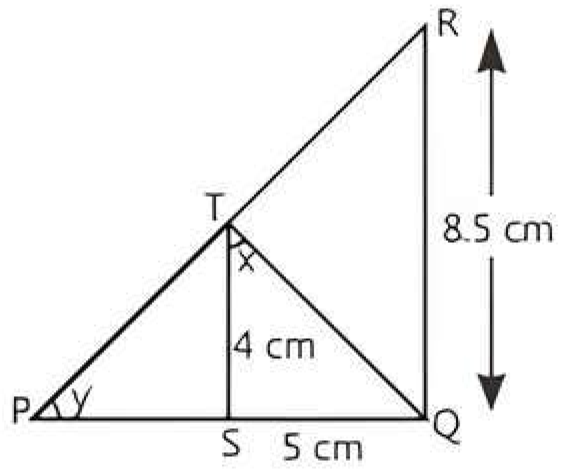
$= 6 + 2 = 8$

Hence, given statement is true.

Case Study Based Questions

Case Study 1

Anika is studying in X standard. She is making a figure to understand trigonometric ratio shown as below.



In ΔPQR, $\angle Q$ is a right angle, ΔQTR is right-angled at T and ΔQST is right-angled at S, $PQ = 12$ cm, $QR = 8.5$ cm, $ST = 4$ cm, $SQ = 5$ cm, $\angle QTS = x$ and $\angle TPQ = y$.

Based on the above information, solve the following questions:

Q 1. The length of PT is:

a. 8 cm b. $\sqrt{65}$ cm

c. 7.5 cm d. $\sqrt{69}$ cm

Q 2. The value of tan *x* is:

a. $\frac{7.5}{13}$ b. $\frac{5}{4}$

c. $\frac{4}{5}$ d. $\frac{13}{7.5}$

Q 3. The value of sec *x* is:

a. $\frac{\sqrt{91}}{6}$ b. $\frac{\sqrt{71}}{6}$

c. $\frac{\sqrt{41}}{4}$ d. $\frac{\sqrt{31}}{5}$

Q 4. The value of sin *y* is:

a. $\frac{4}{\sqrt{65}}$ b. $\frac{4}{7}$

c. $\frac{7}{4}$ d. $\frac{\sqrt{65}}{7}$

Q 5. The value of cot *y* is:

a. $\frac{7}{4}$ b. $\frac{4}{7}$

c. $\frac{\sqrt{65}}{4}$ d. $\frac{\sqrt{65}}{7}$

Solutions

1. We have, $PS = PQ - SQ = 12 - 5 = 7$ cm

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔPST,

$(PT)^2 = (PS)^2 + (ST)^2$ (By Pythagoras theorem)

$= (7)^2 + (4)^2 = 49 + 16 = 65$

$\Rightarrow PT = \sqrt{65}$ cm

So, option (b) is correct.

2. In right-angled ΔTSQ,

$$\tan x = \frac{\text{Perpendicular}}{\text{Base}} = \frac{SQ}{TS} = \frac{5}{4}$$

So, option (b) is correct.

3. We know the identity.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{5}{4}\right)^2 = 1 + \frac{25}{16} = \frac{41}{16}$$

$$\Rightarrow \quad \sec x = \sqrt{\frac{41}{16}} = \frac{\sqrt{41}}{4}$$

So, option (c) is correct.

4. In right-angled ΔTSP,

$$\sin y = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{TS}{PT} = \frac{4}{\sqrt{65}}$$

So, option (a) is correct.

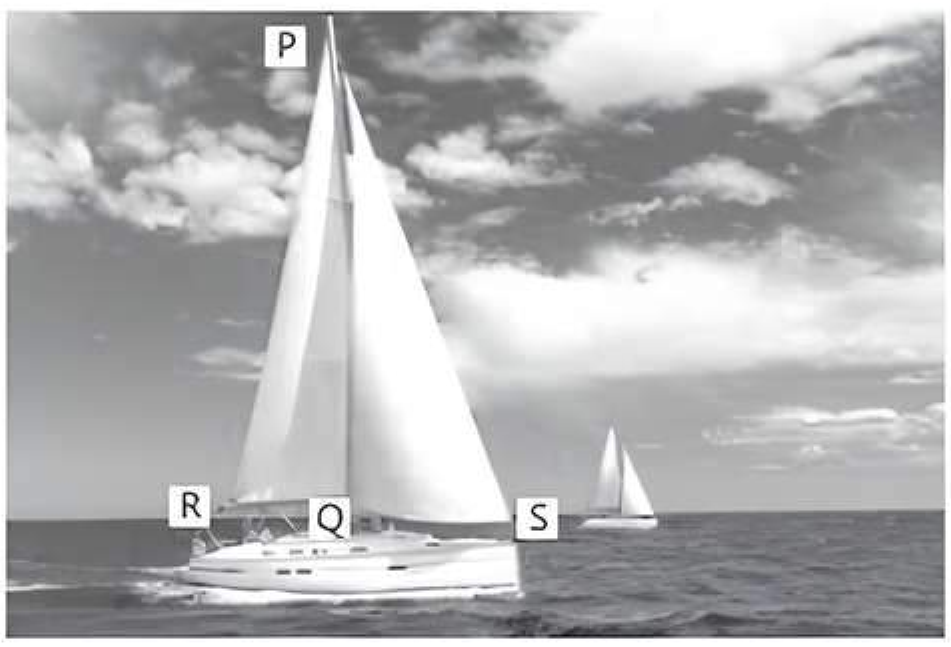
5. In right angled ΔTSP

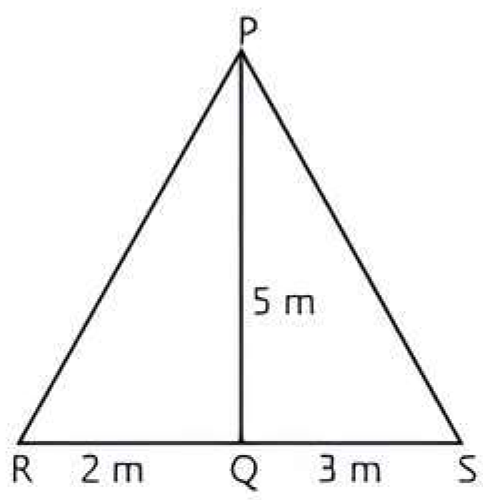
$$\cot y = \frac{\text{Base}}{\text{Perpendicular}} = \frac{PS}{TS} = \frac{7}{4}$$

So, option (a) is correct.

Case Study 2

A sailing boat with triangular masts is as shown below. Two right triangles can be observed Triangles PQR and PQS, both right-angled at Q. The distance QR = 2 m and QS = 3 m and height PQ = 5 m.





Based on the above information, solve the following questions:

Q 1. The value of sec S is:

a. $\frac{\sqrt{34}}{5}$ b. $\frac{\sqrt{34}}{3}$

c. $\frac{5}{3}$ d. $\frac{3}{\sqrt{34}}$

Q 2. The value of cosec R is:

a. $\frac{\sqrt{29}}{5}$ b. $\frac{\sqrt{29}}{2}$

c. $\frac{2}{5}$ d. $\frac{5}{\sqrt{29}}$

Q 3. The value of tan S + cot R is:

a. $\frac{9}{4}$ b. $\frac{5}{3}$ c. $\frac{31}{15}$ d. $\frac{9}{5}$

Q 4. The value of $\sin^2 R - \cos^2 S$ is:

a. 0 b. 1

c. $\frac{97}{85}$ d. $\frac{589}{986}$

Q 5. The value of $\sin^2 S + \cos^2 R$ is:

a. 0 b. 1

c. $\frac{97}{85}$ d. $\frac{861}{986}$

Solutions

1. In right-angled ΔPQS

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$(PS)^2 = (SQ)^2 + (PQ)^2 = (3)^2 + (5)^2 = 9 + 25 = 34$$

(by Pythagoras theorem)

$$\Rightarrow \quad PS = \sqrt{34} \text{ m}$$

$\therefore$ In right-angled ΔPQS,

$$\sec S = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{PS}{SQ} = \frac{\sqrt{34}}{3}$$

So, option (b) is correct.

2. In right-angled ΔPQR

$$(PR)^2 = (PQ)^2 + (QR)^2$$

(by Pythagoras theorem)

$$= (5)^2 + (2)^2 = 25 + 4$$

$$= 29$$

$$\Rightarrow \quad PR = \sqrt{29} \text{ m}$$

$\therefore$ In right-angled ΔPQR,

$$\text{cosec } R = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{PR}{PQ} = \frac{\sqrt{29}}{5}$$

So, option (a) is correct.

3. Use the identity,

$$1 + \tan^2 S = \sec^2 S$$

$$\Rightarrow \quad \tan S = \sqrt{\sec^2 S - 1} = \sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - 1}$$

(from part 1)

$$= \sqrt{\frac{34}{9} - 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

Use the identity, $1 + \cot^2 R = \text{cosec}^2 R$

$$\Rightarrow \quad \cot R = \sqrt{\text{cosec}^2 R - 1} = \sqrt{\left(\frac{\sqrt{29}}{5}\right)^2 - 1}$$

(from part 2)

$$= \sqrt{\frac{29}{25} - 1} = \sqrt{\frac{4}{25}} = \frac{2}{5}$$

$$\therefore \quad \tan S + \cot R = \frac{5}{3} + \frac{2}{5} = \frac{25+6}{15} = \frac{31}{15}$$

So, option (c) is correct.

4. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$$

TiP

$\cos\theta = \frac{1}{\sec\theta}$, $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

From part (2), $\operatorname{cosec} R = \frac{\sqrt{29}}{5}$

$$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$$

$$\therefore \sin^2 R - \cos^2 S = \left(\frac{5}{\sqrt{29}}\right)^2 - \left(\frac{3}{\sqrt{34}}\right)^2 = \frac{25}{29} - \frac{9}{34}$$

$$= \frac{850 - 261}{986} = \frac{589}{986}$$

So, option (d) is correct.

5. From part (1), $\sec S = \frac{\sqrt{34}}{3}$

$$\Rightarrow \quad \cos S = \frac{3}{\sqrt{34}}$$

$$\sin S = \sqrt{1 - \cos^2 S} = \sqrt{1 - \frac{9}{34}} = \sqrt{\frac{25}{34}} = \frac{5}{\sqrt{34}}$$

From part (2), $\operatorname{cosec} R = \frac{\sqrt{29}}{5}$

$$\Rightarrow \quad \sin R = \frac{5}{\sqrt{29}}$$

$$\therefore \quad \cos R = \sqrt{1 - \sin^2 R} = \sqrt{1 - \frac{25}{29}} = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$$

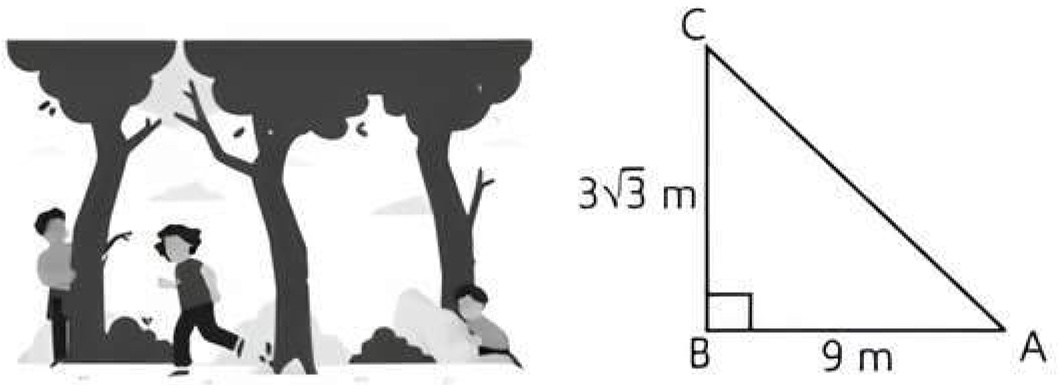
$$\sin^2 S + \cos^2 R = \left(\frac{5}{\sqrt{34}}\right)^2 + \left(\frac{2}{\sqrt{29}}\right)^2 = \frac{25}{34} + \frac{4}{29}$$

$$= \frac{725 + 136}{986} = \frac{861}{986}$$

So, option (d) is correct.

Case Study 3

Three friends—Sanjeev, Amit and Digvijay are playing hide and seek in a park. Sanjeev and Amit were supposed to hide and Digvijay had to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle such that AB = 9 m, BC = $3\sqrt{3}$ m and $\angle B = 90^\circ$.



Based on the above information, solve the following questions:

Q 1. Find the measure of ∠A by using trigonometric ratio.

Q 2. Find the measure of ∠C by using trigonometric ratio.

Q 3. Find the length of AC.

Q 4. Find the value of cos 2A.

Or

Find the value of $\sin\left(\frac{C}{2}\right)$.

Solutions

1. We have, AB = 9 m, BC = $3\sqrt{3}$ m

In right △ABC, we have

$$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan A = \tan 30^\circ \Rightarrow \angle A = 30^\circ$$

2. In right △ABC,

We have, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$$\Rightarrow \quad \tan C = \tan 60^\circ \Rightarrow \angle C = 60^\circ$$

3. In right △ABC, $\sin A = \frac{BC}{AC}$

$$\Rightarrow \quad \sin 30^\circ = \frac{BC}{AC} \quad \text{[from part (1)]}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}\text{ m}$$

4. $\because \quad \angle A = 30^\circ$ [from part (1)]

$$\therefore \quad \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

Or

$$\angle C = 60^\circ$$

$$\therefore \quad \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \frac{1}{2}$$

Case Study 4

Soniya and her father went to her friend Ruhi to enjoy party. When they reached Ruhi's place, Soniya saw the roof of the house, which was triangular in shape. She imagined the dimensions of the roof which is as given in the figure.

Based on the above information, solve the following questions:

Q 1. If D is the mid-point of AC, then find BD.

Q 2. Find the measure of ∠A and ∠C.

Q 3. Find the value of sin A + cos C.

Or

Find the value of $\tan^2 C + \tan^2 A$.

Solutions

1. We have, $AB = BC = 6\sqrt{2}$ m and AC = 12 m

$\because$ D is the mid–point of AC.

$\therefore \quad AD = DC = \frac{12}{2} = 6$ m

In right-angled ΔADB, use Pythagoras theorem

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad BD^2 = (6\sqrt{2})^2 - 6^2$$

$$BD^2 = 72 - 36 = 36$$

$$\Rightarrow \quad BD = 6 \text{ m}$$

2. In right ΔADB, $\sin A = \frac{BD}{AB} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$ [from part (1)]

$\Rightarrow \quad \sin A = \sin 45° \quad \Rightarrow \angle A = 45°$

In right ΔBDC, $\tan C = \frac{BD}{DC} = \frac{6}{6}$

$\Rightarrow \quad \tan C = 1 = \tan 45° \Rightarrow \angle C = 45°$

3. Here, $\sin A = \frac{1}{\sqrt{2}}$ and $\cos C = \cos 45° = \frac{1}{\sqrt{2}}$

$\therefore \sin A + \cos C = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

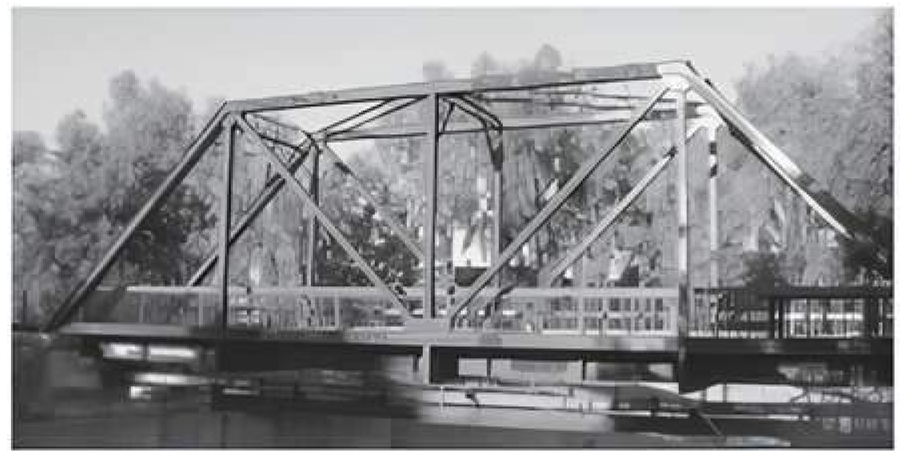
Or

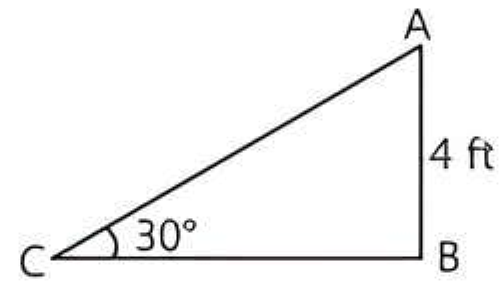
Here, tan C = 1 and tan A = tan 45° = 1

$\Rightarrow \quad \tan^2 C + \tan^2 A = 1 + 1 = 2$

Case Study 5

In structural design, a structure is composed of triangles that are interconnecting. A truss a series of triangle in same plane end is one of the major types of engineering structures and is especially used in the design of bridges and buildings. Trusses are designed to support loads, such as the weight of people. A truss is exclusively made of long, straight members connected by joints at the end of each member.





This is a single repeating triangle in a truss system.

Based on the above information, solve the following questions:

Q 1. In the above triangle, what is the length of AC?

Q 2. In the above triangle, what is the length of BC?

Q 3. If sin A = sin C, what will be the length of BC?

Or

If the length of AB doubles, what will happen the length of AC?

Solutions

1. In right angled ΔABC,

$$\sin 30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{4}{AC}$$

$$\Rightarrow \quad AC = 8 \text{ ft}$$

2. In right-angled ΔABC,

$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{BC} \Rightarrow BC = 4\sqrt{3} \text{ ft}$$

3. Given, $\quad \sin A = \sin C$

In right-angled Δ ABC,

$$\frac{BC}{AC} = \frac{AB}{AC} \Rightarrow BC = AB = 4 \text{ ft}$$

Or

Given, AB = 2 × 4 = 8 ft

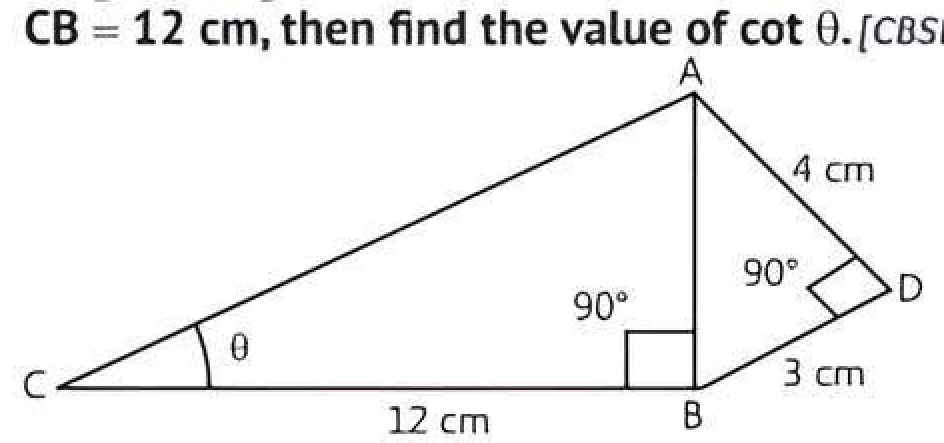
$\therefore$ In right ΔABC, $\sin 30° = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{2} = \frac{8}{AC} \Rightarrow AC = 16 \text{ ft}$$

So, AC doubles the original length.

Very Short Answer Type Questions

Q 1. In given figure, if AD = 4 cm, BD = 3 cm and CB = 12 cm, then find the value of cot θ. *[CBSE 2016]*



Q 2. If $\sqrt{3}\sin\theta = \cos\theta$, then find the value of

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$$ [CBSE 2015]

Q 3. Evaluate:

$$\frac{5}{\cos^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°.$$

[CBSE 2023]

Q 4. Evaluate $2\sec^2\theta + 3\operatorname{cosec}^2\theta - 2\sin\theta\cos\theta$ if $\theta = 45$. [CBSE 2023]

Q 5. If $\sin x + \cos y = 1$, $x = 30°$ and y is an acute angle, find the value of y. [CBSE 2019]

Q 6. Find the value of x:

$$2\operatorname{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$$

[CBSE SQP 2023-24]

Q 7. If $4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of p. [CBSE 2023]

Q 8. If $\sin\theta - \cos\theta = 0$, then find the value of $\sin^4\theta + \cos^4\theta$. [CBSE 2023, 17]

Q 9. If θ is an acute angle and $\sin\theta = \cos\theta$, find the value of $\tan^2\theta + \cot^2\theta - 2$. [CBSE 2023]

Q 10. Find the value of $(\operatorname{cosec}^2\theta - 1)\cdot\tan^2\theta$. [CBSE 2016]

Q 11. If $\sec\theta + \tan\theta = 7$, then evaluate $\sec\theta - \tan\theta$.

Q 12. If $\tan\theta = \frac{a}{x}$, find the value of $\frac{x}{\sqrt{a^2 + x^2}}$.

Q 13. If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$. [CBSE 2023]

Short Answer Type-I Questions

Q 1. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

[CBSE 2016]

Q 2. If $\tan(A+B) = \sqrt{3}$ and $(A-B) = \frac{1}{\sqrt{3}}$;

$0° < A + B \le 90°$, $A > B$, then find A and B.

[NCERT EXERCISE; CBSE SQP 2023-24; CBSE 2016]

Q 3. If $\sin(A+B) = 1$ and $\cos(A-B) = \sqrt{3}/2$, $< A + B \le 90°, 0°$ and $A > B$, then find the measures of angles A and B. [CBSE SQP 2022-23]

Q 4. Prove that $1 + \frac{\cot^2\alpha}{1 + \operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$.

[NCERT EXEMPLAR; CBSE 2020]

Q 5. Prove that $2\cos^2\theta + \frac{2}{1+\cot^2\theta} = 2$. [V. Imp.]

Q 6. Find an acute angle θ when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Q 7. Prove that: $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$. [CBSE 2023]

Q 8. Prove that $(\sec^4\theta - \sec^2\theta) = (\tan^2\theta + \tan^4\theta)$.

[NCERT EXEMPLAR; V. Imp., CBSE 2020]

Q 9. Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$.

[NCERT EXERCISE; CBSE 2015]

Q 10. Express sin A and cos A in terms of cot A.

[NCERT EXERCISE; CBSE 2015]

Q 11. Prove that: $\sec A(1 - \sin A)(\sec A + \tan A) = 1$.

[CBSE 2023]

Short Answer Type-II Questions

Q 1. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

[CBSE 2018]

Q 2. If $\sin\theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cdot\cos\theta} \times \frac{1}{\tan^2\theta}.$$

[CBSE 2015]

Q 3. Prove that $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$. [CBSE 2023]

Q 4. Prove that $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$.

[V. Imp.]

Q 5. Prove that

$$\left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$$

Q 6. Prove that

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

[NCERT EXERCISE; CBSE 2016, 19]

Q 7. Prove that $(\operatorname{cosec}\theta + \cot\theta)^2 = \frac{\sec\theta + 1}{\sec\theta - 1}$.

[CBSE 2016]

Q 8. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

[NCERT EXEMPLAR; V. Imp.]

Q 9. If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$.

Q 10. Prove that:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$$ [CBSE 2023]

Q 11. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$. [NCERT EXEMPLAR; CBSE 2020; CBSE SQP 2023-24]

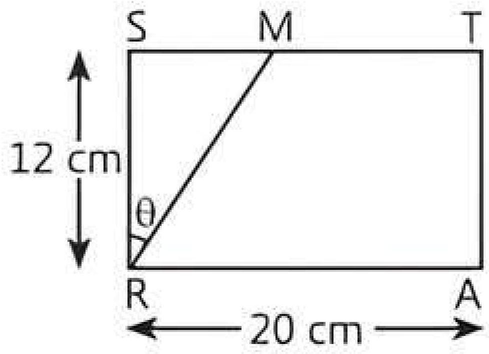
Q 12. Prove the following that:

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\ \text{cosec}\theta-2\sin\theta\cos\theta.$$

[CBSE SQP 2022-23]

Long Answer Type Questions

Q 1. In the given figure, STAR is a rectangle with SR = 12 cm and AR = 20 cm. Line segment RM is drawn making an angle of θ with SR, intersecting ST at M.



Based on the given figure, solve the following questions:

(i) Find the value of θ if $2\sin(60°-\theta)=1$, where $0°<\theta<90°$.

(ii) Find the lengths of RM and MT.

(iii) Evaluate using value of θ obtained in (i)

$$3\cot^2(3\theta)+\sec^2(2\theta)$$

Q 2. If $m=\cos\theta-\sin\theta$ and $n=\cos\theta+\sin\theta$, then show that $\sqrt{\frac{m}{n}}+\sqrt{\frac{n}{m}}=\frac{2}{\sqrt{1-\tan^2}}\theta$. *[CBSE 2016]*

Q 3. If $\sqrt{3}\cot^2\theta-4\cot\theta+\sqrt{3}=0$, then find the value of $\cot^2\theta+\tan^2\theta$.

Q 4. In an acute angled triangle ABC, if $\sin(A+B-C)=\frac{1}{2}$ and $\cos(B+C-A)=\frac{1}{\sqrt{2}}$, find $\angle A$, $\angle B$ and $\angle C$.

Q 5. Prove that $\frac{\cos^2}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cdot\cos\theta.$

[CBSE 2017]

Q 6. Prove that $\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta}=2+\frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}$.

Q 7. Prove that

$\sin^2\theta\tan\theta+\cos^2\theta\cot\theta+2\sin\theta\cos\theta=\tan\theta+\cot\theta.$

[CBSE 2016]

Q 8. Prove that $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}=\frac{1}{\sec A-\tan A}$.

[NCERT EXERCISE, CBSE 2019]

Q 9. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$,

prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2.$ *[CBSE 2017]*

Q 10. Prove the following trigonometric identities:

$\sin A(1+\tan A)+\cos A(1+\text{cost}\,A)=\sec A+\text{cosec}\,A.$

[CBSE 2015]

Q 11. If $\sin\theta+\cos\theta=m$ and $\sec\theta+\text{cosec}\theta=n$, then prove that $n(m^2-1)=2m$. *[NCERT EXEMPLAR]*

Q 12. If $\sec\theta+\tan\theta=m$, show that $\frac{m^2-1}{m^2+1}=\sin\theta$.

[CBSE 2019, 16]

Q 13. If $\tan A=n\tan B$ and $\sin A=m\sin B$, then prove that $\cos^2A=\frac{m^2-1}{n^2-1}$. *[CBSE 2016]*

Solutions

Very Short Answer Type Questions

1. In right-angled ΔADB.

TiP

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$AB^2=BD^2+AD^2$ (by Pythagoras theorem)

$=(3)^2+(4)^2=9+16=25$

$\therefore\quad AB=5$ cm

Now. In right-angled ΔABC.

$$\cot\theta=\frac{BC}{AB}=\frac{12}{5}.$$

2. $\frac{3\cos^2\theta+2\cos\theta}{3\cos\theta+2}$

$$=\frac{3\times(\sqrt{3}\sin\theta)^2+2\times\sqrt{3}\sin\theta}{3\times\sqrt{3}\sin\theta+2}$$

$(\because\cos\theta=\sqrt{3}\sin\theta)$

$$=\frac{3\times3\sin^2\theta+2\sqrt{3}\sin\theta}{3\sqrt{3}\sin\theta+2}$$

$$=\frac{\sqrt{3}\sin\theta(3\sqrt{3}\sin\theta+2)}{(3\sqrt{3}\sin\theta+2)}$$

3. $\frac{5}{\cot^2 30°}+\frac{1}{\sin^2 60°}-\cot^2 45°+2\sin^2 90°$

$$=\frac{5}{(\sqrt{3})^2}+\frac{1}{(\sqrt{3}/2)^2}-(1)^2+2\times(1)^2$$

$$=\frac{5}{3}+\frac{1}{(3/4)}-1+2=\frac{5}{3}+\frac{4}{3}+1=\frac{9}{3}+1=3+1=4$$

4. $2\sec^2\theta+3\,\text{cosec}^2\theta-2\sin\theta\cdot\cos\theta$

$=2\sec^2 45°+3\,\text{cosec}^2 45°-2\sin 45°\cdot\cos 45°$

(at $\theta=45°$)

$$=2(\sqrt{2})^2+3(\sqrt{2})^2-2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$$

$$=2\times2+3\times2-2\times\frac{1}{2}=4+6-1=10-1=9.$$

5. Given, $\sin x + \cos y = 1$

Put $x = 30°$,

$\sin 30° + \cos y = 1 \Rightarrow \cos y = 1 - \frac{1}{2}$ $(\because \sin 30° = 1/2)$

$\Rightarrow \cos y = \frac{1}{2} \Rightarrow y = 60°$ $(\because \cos 60° = 1/2)$

6. $2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$

$\Rightarrow 2\times(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\times\left(\frac{1}{\sqrt{3}}\right)^2 = 10$

$\Rightarrow 2\times 4 + x\times\frac{3}{4} - \frac{3}{4}\times\frac{1}{3} = 10$

$\Rightarrow 8 + \frac{3x}{4} - \frac{1}{4} = 10$

$\Rightarrow \frac{3x}{4} = 10 - 8 + \frac{1}{4}$

$= 2 + \frac{1}{4} = \frac{9}{4}$

$\Rightarrow x = \frac{9}{3} = 3.$

7. $4\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$

$\Rightarrow 4\times(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + p = \frac{3}{4}$

$\Rightarrow 4\times 1 - 4 + \frac{3}{4} + p = \frac{3}{4}$

$\Rightarrow p = \frac{3}{4} - \frac{3}{4} + 4 - 4 = 0$

8. Given, $\sin\theta - \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 45° + \cos^4 45°$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$(\because \tan 45° = 1 \text{ and } \sin 45° = \cos 45° = 1/\sqrt{2})$

9. Given, $\sin\theta = \cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°.$

$\therefore \tan^2\theta + \cot^2\theta - 2 = \tan^2 45° + \cot^2 45° - 2$

$= (1)^2 + (1)^2 - 2$

$= 1 + 1 - 2 = 2 - 2 = 0.$

10. $(\text{cosec}^2\theta - 1)\cdot\tan^2\theta = \cot^2\theta\cdot\tan^2\theta$

$= \cot^2\theta\cdot\frac{1}{\cot^2\theta} = 1$

$\left(\because \text{cosec}^2\theta - 1 = \cot^2\theta \text{ and } \tan\theta = \frac{1}{\cot\theta}\right)$

11. Given, $\sec\theta + \tan\theta = \frac{7}{1}$

$\Rightarrow \frac{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = \frac{7}{1}$

$\Rightarrow \sec\theta - \tan\theta = \frac{\sec^2\theta - \tan^2\theta}{7}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$\therefore \sec\theta - \tan\theta = \frac{1}{7}$ $(\because \sec^2\theta - \tan^2\theta = 1)$

12. $\frac{x}{\sqrt{a^2 + x^2}} = \frac{x}{x\sqrt{\left(\frac{a}{x}\right)^2 + 1}} = \frac{1}{\sqrt{\left(\frac{a}{x}\right)^2 + 1}}$

$= \frac{1}{\sqrt{\tan^2\theta + 1}}$ $\left(\because \tan\theta = \frac{a}{x}\right)$

$= \frac{1}{\sqrt{\sec^2\theta}} = \frac{1}{\sec\theta} = \cos\theta$ $(\because \tan^2\theta + 1 = \sec^2\theta)$

13. Given, $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$ $(\because \sin^2 A + \cos^2 A = 1)$

Squaring on both sides, we get

$\cos^2 A = \sin^4 A$

$\Rightarrow 1 - \sin^2 A = \sin^4 A$ $(\because \cos^2 A = 1 - \sin^2 A)$

$\Rightarrow \sin^2 A + \sin^4 A = 1.$

Short Answer Type-I Questions

1. Given expression $= \frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$

$= \frac{2(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)2(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \cot^2\theta = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$ $\left(\because \cot\theta = \frac{15}{8}\right)$

2. Given, $\tan(A + B) = \sqrt{3}$

$\Rightarrow \tan(A + B) = \tan 60°$ $(\because \tan 60° = \sqrt{3})$

$\Rightarrow A + B = 60°$ ...(1)

Again, $\tan(A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A - B) = \tan 30°$ $(\because \tan 30° = 1/\sqrt{3})$

$\Rightarrow A - B = 30°$ ...(2)

Adding eqs. (1) and (2), we get

$2A = 90° \Rightarrow A = 45°$

From eq. (1), we get

$45° + B = 60° \Rightarrow B = 15°$

Hence, $\angle A = 45°$ and $\angle B = 15°$

COMMON ERROR

Some students confused between values of tan 30° and tan 60°. They take wrong values as tan 30° = $\sqrt{3}$ and tan 60° = $\frac{1}{\sqrt{3}}$.

3. Given,

$$\sin(A+B) = 1 = \sin 90°$$

So, $A + B = 90°$ ...(1)

and $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30°$ ...(2)

$$A - B = 30°$$

Adding eqs. (1) and (2),

$$2\angle A = 120° \Rightarrow \angle A = 60°$$

Put $\angle A = 60°$ in eq. (1),

$$\angle B = 30°$$

4. LHS $= 1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}$

$= 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$ $(\because \operatorname{cosec}^2\theta + \cot^2\theta = 1)$

$= 1 + \frac{(\operatorname{cosec}\alpha+1)(\operatorname{cosec}\alpha-1)}{(1+\operatorname{cosec}\alpha)}$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= 1 + \operatorname{cosec}\alpha - 1 = \operatorname{cosec}\alpha$

= RHS **Hence proved.**

COMMON ERROR

Sometimes students don't apply this formula: $(a^2 - b^2) = (a+b)(a-b)$. They directly simplify equation which leads to incorrect result.

5. LHS $= 2\cos^2\theta + \frac{2}{1+\cot^2\theta}$

$= 2\cos^2\theta + \frac{2}{\operatorname{cosec}^2\theta}$ $(\because 1+\cot^2\theta = \operatorname{cosec}^2\theta)$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta)$ $(\because \cos^2\theta + \sin^2\theta = 1)$

$= 2 \times 1 = 2 = $ RHS **Hence proved.**

6. Given, $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Dividing the numerator and denominator of LHS by $\cos\theta$.

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Comparing both sides, $\tan\theta = \sqrt{3}$

or $\theta = 60°$

7. L.H.S. $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$= \frac{\sin A\{1 - 2\sin^2 A\}}{\cos A\{2\cos^2 A - 1\}}$ $(\because \sin^2 A + \cos^2 A = 1)$

$= \frac{\sin A\{1-2\sin^2 A\}}{\cos A\{2(1-\sin^2 A) - 1\}}$

$= \frac{\sin A(1-2\sin^2 A)}{\cos A\{2 - 2\sin^2 A - 1\}} = \frac{\sin A(1-2\sin^2 A)}{\cos A(1-2\sin^2 A)}$

$= \frac{\sin A}{\cos A} = \tan A = $ R.H.S.

8. LHS $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1)$

TR!CK

$\because$ $\sec^2\theta - \tan^2\theta = 1$

$\therefore$ $1 + \tan^2\theta = \sec^2\theta$

or $\tan^2\theta = \sec^2\theta - 1$

$= (1+\tan^2\theta)\tan^2\theta$

$= \tan^2\theta + \tan^4\theta = $ RHS **Hence proved.**

9. LHS $= \sqrt{\frac{1+\sin A}{1-\sin A}}$

TiP

Follow step-by-step simplification to avoid errors.

$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}}$ $(\because \sin^2 A + \cos^2 A = 1)$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

= RHS **Hence proved.**

10. Since, $1 + \cot^2 A = \operatorname{cosec}^2 A$

$\Rightarrow$ $\operatorname{cosec} A = \sqrt{1+\cot^2 A}$

$\Rightarrow$ $\frac{1}{\sin A} = \sqrt{1+\cot^2 A}$

$\Rightarrow$ $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

and $\cos A = \sqrt{1-\sin^2 A}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \sqrt{1 - \frac{1}{1+\cot^2 A}} = \sqrt{\frac{1+\cot^2 A - 1}{1+\cot^2 A}}$

$= \frac{\cot A}{\sqrt{1+\cot^2 A}}$

11. L.H.S. $= \sec A(1-\sin A)(\sec A + \tan A)$

$= \sec A \cdot (1-\sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \sec A \cdot (1-\sin A) \cdot \frac{(1+\sin A)}{\cos A}$

$= \sec A \cdot \frac{(1-\sin^2 A)}{\cos A}$ $[\because (a-b)(a+b) = a^2 - b^2]$

$= \sec A \cdot \frac{\cos^2 A}{\cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$

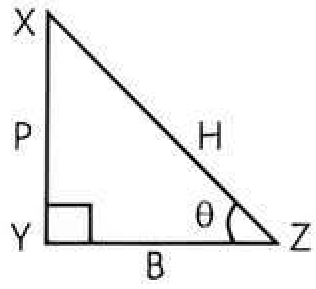
$= \frac{1}{\cos A} \times \cos A = 1 = $ R.H.S.

Short Answer Type-II Questions

1. Let triangle XYZ is a right-angled triangle in which $\angle Y = 90°$.

Given, $4 \tan\theta = 3$

$\Rightarrow \tan\theta = \frac{3}{4} = \frac{P}{B} = \frac{XY}{YZ}$



Let $P = 3k$ and $B = 4k$, where k is a positive number.

In right-angled ΔXYZ,

$(XZ)^2 = (XY)^2 + (YZ)^2$ (by Pythagoras theorem)

$\Rightarrow H^2 = P^2 + B^2 = (3k)^2 + (4k)^2$

$= 9k^2 + 16k^2 = 25k^2$

$\Rightarrow H = 5k$

TR!CK

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

So, $\sin\theta = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$ and $\cos\theta = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$

$$\therefore \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4\left(\frac{3}{5}\right) - \frac{4}{5} + 1}{4\left(\frac{3}{5}\right) + \frac{4}{5} - 1} = \frac{\frac{12-4+5}{5}}{\frac{12+4-5}{5}}$$

$$= \frac{12-4+5}{12+4-5} = \frac{17-4}{16-5} = \frac{13}{11}$$

2. Given, $\sin\theta = \frac{12}{13}$

TiP

Here, we have given only the value of sin θ. So, we have to convert the given expression into sin θ with the help of suitable identities.

$$\therefore \frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$$

$$= \frac{\sin^2\theta - (1-\sin^2\theta)}{2\sin\theta\cos\theta} \times \frac{\cos^2}{\sin^2\theta}$$

$$= \frac{2\sin^2\theta - 1}{2\sin^3\theta}.\cos\theta = \frac{2\sin^2\theta - 1}{2\sin^3\theta}.\sqrt{1-\sin^2\theta}$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$$= \frac{2\left(\frac{12}{13}\right)^2 - 1}{2\left(\frac{12}{13}\right)^3}.\sqrt{1-\left(\frac{12}{13}\right)^2} = \frac{2\times\frac{144}{169} - 1}{2\times\frac{12}{13}\times\frac{144}{169}} \times \sqrt{1-\frac{144}{169}}$$

$$= \frac{\frac{288-169}{169}}{\frac{24}{13}\times\frac{144}{169}} \times \sqrt{\frac{25}{169}} = \frac{119}{169} \times \frac{13\times169}{24\times144} \times \frac{5}{13}$$

$$= \frac{119\times5}{24\times144} = \frac{595}{3456}.$$

3. L.H.S. $= \frac{1+\sec A}{\sec A} = \frac{1+1/\cos A}{1/\cos A}$

$$= \frac{\cos A + 1}{\cos A} \times \cos A = (1+\cos A) \times \frac{(1-\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} \quad (\because \sin^2 A + \cos^2 A = 1)$$

= R.H.S.

4. LHS $= \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \cos\theta\sin\theta)}{(\cos\theta + \sin\theta)}$$

$$+ \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)}{(\cos\theta - \sin\theta)}$$

$= (1 - \cos\theta\sin\theta) + (1 + \cos\theta\sin\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1 + 1 = 2 =$ RHS **Hence proved.**

COMMON ERR(!)R

Sometimes students don't apply these formulae:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

They directly simplify equation which leads to incorrect result.

5. LHS $= \left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right)$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$= (1 + \cot^2 A)(1 + \tan^2 A)$ $(\because \tan A \cdot \cot A = 1)$

$= \text{cosec}^2 A \cdot \sec^2 A = \frac{1}{\sin^2 A} \cdot \frac{1}{\cos^2 A}$

$(\because \text{cosec}^2\theta - \cot^2\theta = 1$ and $\sec^2\theta - \tan^2\theta = 1)$

$= \frac{1}{\sin^2 A(1 - \sin^2 A)}$ $(\because \sin^2 A + \cos^2 A = 1)$

$= \frac{1}{\sin^2 A - \sin^4 A} =$ RHS **Hence proved.**

6. LHS $= (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta$

$+ \sec^2\theta + 2\cos\theta\sec\theta$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$

$+ 2\sin\theta\left(\frac{1}{\sin\theta}\right) + \cos\theta\left(\frac{1}{\cos\theta}\right)$

$= 1 + (1 + \cot^2\theta + 1 + \tan^2\theta) + 2 + 2$

$(\because \text{cosec}^2\theta = 1 + \cot^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta)$

$= 7 + \tan^2\theta + \cot^2\theta =$ RHS **Hence proved.**

7. LHS $= (\text{cosec}\,\theta + \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin^2} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)} \quad (\because a^2 - b^2 = (a+b)(a-b))$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}} \quad \left(\because \cos\theta = \frac{1}{\sec\theta}\right)$$

$$= \frac{\frac{\sec\theta+1}{\sec\theta}}{\frac{\sec\theta-1}{\sec\theta}} = \frac{\sec\theta+1}{\sec\theta-1} = \text{RHS}$$

Hence proved.

8. LHS $= \sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

TiP

Adequate practice of identities is necessary to avoid errors in simplification.

$= (\sin^2\theta + \cos^2\theta)\,[(\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$[\because a^3 + b^3 = (a+b)(a^2+b^2-ab)]$

$= 1 \times [(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta)$

$- 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

(on adding and subtracting $2\sin^2\theta\cos^2\theta$)

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= 1^2 - 3\sin^2\theta\cos^2\theta \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 1 - 3\sin^2\theta\cos^2\theta = \text{RHS}$ **Hence proved**

9. Given, $a\cos\theta - b\sin\theta = c$

TiP

Memorize the all identities of trigonometric ratios properly and do practice more.

On squaring both sides, we get

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\sin\theta\cos\theta = c^2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow a^2 - a^2\sin^2\theta + b^2 - b^2\cos^2\theta - 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\therefore a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$ **Hence proved.**

10. L.H.S. $= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^2\theta + 1 + \cos\theta^2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{(1+\cos\theta)\cdot\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = 2\cdot\frac{1}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{R.H.S.}$

11. Given, $1 + \sin^2\theta = 3\sin\theta\cdot\cos\theta$

Dividing on both sides by $\cos^2\theta$, we get

$\frac{1+\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cdot\cos\theta}{\cos^2\theta}$

$\Rightarrow \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 3\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\cos\theta}$

$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 3\tan\theta \quad (\because \sec^2\theta = 1 + \tan^2\theta)$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

TR!CK

$\because 2 \times 1 = 2$

$\therefore$ The sum of -2 and -1 is -3 i.e., middle term.

$\Rightarrow 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$\Rightarrow 2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - 1)(2\tan\theta - 1) = 0$

$\therefore \tan\theta = 1$ or $\frac{1}{2}$ **Hence proved.**

12. LHS $= \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$

$= \frac{\sin^3\theta/\cos^3\theta}{1+\sin^2\theta/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{1+\cos^2\theta/\sin^2\theta}$

$= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta+\sin^2\theta)/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta+\cos^2\theta)/\sin^2\theta}$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} = \frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$

$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \sec\theta\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$

$= \text{RHS}$ **Hence proved.**

Long Answer Type Questions

1. (i) Given, $2\sin(60^\circ - \theta) = 1$

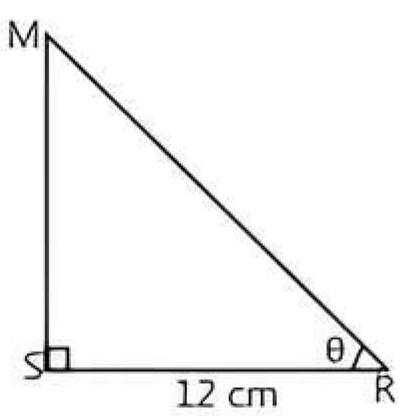
$\Rightarrow \sin(60^\circ - \theta) = \frac{1}{2}$

$\Rightarrow \sin(60^\circ - \theta) = \sin 30^\circ$

$\Rightarrow 60^\circ - \theta = 30^\circ$

$\Rightarrow \theta = 30^\circ$

(ii) In right-angled ΔMSR,



$\cos 30^\circ = \frac{SR}{RM}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{RM}$

$\Rightarrow RM = \frac{24}{\sqrt{3}} = \frac{24}{3}\times\sqrt{3} = 8\sqrt{3}$ cm

Also, $\tan 30^\circ = \frac{SM}{SR} = \frac{SM}{12}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{SM}{12}$

$\Rightarrow \quad SM = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ cm

$\therefore \quad ST = SM + MT$

$\Rightarrow \quad MT = ST - SM$

$= 20 - 4\sqrt{3} \quad (\because ST = AR = 20 \text{ cm})$

$\Rightarrow \quad MT = 4(5 - \sqrt{3})$ cm

Hence, lengths of RM and MT are $8\sqrt{3}$ cm and $4(5-\sqrt{3})$ cm.

(iii) $3\cot^2(3\theta) + \sec^2(2\theta)$

$= 3\cot^2(3 \times 30^\circ) + \sec^2(2 \times 30^\circ)$

$= 3\cot^2 90^\circ + \sec^2 60^\circ = 3 \times 0 + (2)^2$

$= 0 + 4 = 4$

COMMON ERR(!)R

Sometimes students get confused with the values of trigonometric angles. They substitute wrong values which leads to the wrong result.

2. LHS $= \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$

$= \sqrt{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}} + \sqrt{\frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}}$

$= \frac{\sqrt{(\cos\theta - \sin\theta)^2} + \sqrt{(\cos\theta + \sin\theta)^2}}{\sqrt{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}}$

$= \frac{\cos\theta - \sin\theta + \cos\theta + \sin\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{2\cos\theta}{\sqrt{\cos^2\theta - \sin^2\theta}}$

$= \frac{2\frac{\cos\theta}{\cos\theta}}{\sqrt{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}}}$

(divide numerator and denominator by cos θ)

$= \frac{2}{\sqrt{1 - \tan^2\theta}} = \text{RHS}$ **Hence proved.**

3. Given, $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

TR!CK

$\because \quad \sqrt{3} \times \sqrt{3} = 3$

$\therefore \quad 3 = 3 \times 1$

Here we take 3 and 1 as a factors of 3.

So, middle term $-4 = -3 - 1$.

$\sqrt{3}\cot^2\theta - 3\cot\theta - \cot\theta + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}\cot\theta(\cot\theta - \sqrt{3}) - 1(\cot\theta - \sqrt{3}) = 0$

$\Rightarrow \quad (\cot\theta - \sqrt{3})(\sqrt{3}\cot\theta - 1) = 0$

$\Rightarrow \quad \cot\theta - \sqrt{3} = 0$ or $\cot\theta - \sqrt{3} = 0$

$\Rightarrow \quad \cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

When $\cot^2\theta = \sqrt{3}$, $\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \quad \cot^2 + \tan^2\theta = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3}$

When $\cot\theta = \frac{1}{\sqrt{3}}$, $\tan\theta = \sqrt{3}$

$\therefore \quad \cot^2\theta + \tan^2\theta = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$

$= \frac{1}{3} + 3 = \frac{1+9}{3} = \frac{10}{3}$

4. Given, $\sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$

$\Rightarrow \quad A + B - C = 30^\circ$ ...(1)

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$\Rightarrow \quad B + C - A = 45^\circ$ ...(2)

But in ΔABC, $A + B + C = 180^\circ$ ...(3)

Adding eqs. (1) and (2), we get

$(A + B - C) + (B + C - A) = 30^\circ + 45$

$\Rightarrow \quad 2B = 75^\circ \Rightarrow \quad B = 37.5^\circ$

Adding eqs. (1) and (3), we get

$(A + B - C) + (A + B + C) = 30^\circ + 180^\circ$

$\Rightarrow \quad 2(A + B) = 210^\circ$

$\Rightarrow \quad A + B = 105^\circ$ ...(4)

Putting the value of 'B' in eq. (4), we get

$A + 37.5^\circ = 105^\circ$

$\Rightarrow \quad A = 67.5^\circ$

Again on putting the value of 'A' and 'B' in eq. (3), we get

$67.5^\circ + 37.5^\circ + C = 180^\circ$

$\Rightarrow \quad C = 180^\circ - 105^\circ = 75^\circ$

$\therefore \quad \angle A = 67.5^\circ, \angle B = 37.5^\circ$ and $\angle C = 75^\circ$

5. LHS $= \frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$

TiP

Learn basic identities like $(a-b)^2$, $(a+b)^2$, (a^2-b^2), (a^3-b^3), (a^3+b^3), *etc.*

$= \frac{\cos^2\theta}{1 - \left(\frac{\sin\theta}{\cos\theta}\right)} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \frac{\cos^3\theta - \sin^3\theta}{(\cos\theta - \sin\theta)}$

$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cdot\cos\theta)}{(\cos\theta - \sin\theta)}$

$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= (\sin^2\theta + \cos^2\theta) + \sin\theta \cdot \cos\theta$

$= 1 + \sin\theta \cdot \cos\theta = \text{RHS} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

Hence proved.

COMMON ERROR

Sometimes students don't apply this formula:

$$a^3 - b^3 = (a-b)(a^2+ab+b^2).$$

They directly simplify equation which leads to incorrect result.

6. LHS $= \dfrac{\sin\theta}{\cot\theta + \text{cosec}\theta} = \dfrac{\sin\theta}{\left(\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta}\right)} = \dfrac{\sin^2\theta}{\cos\theta + 1}$

$= \dfrac{1-\cos^2\theta}{1+\cos\theta}$ $\dfrac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)} = 1-\cos\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

RHS $= 2 + \dfrac{\sin\theta}{\cot\theta - \text{cosec}\theta} = 2 + \dfrac{\sin\theta}{\left(\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)}$

$= 2 + \dfrac{\sin^2\theta}{\cos\theta - 1} = 2 + \dfrac{1-\cos^2\theta}{\cos\theta - 1}$

$= \dfrac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)} = 2 - (1+\cos\theta)$

$= 2 - 1 - \cos\theta = 1 - \cos\theta$

So, LHS = RHS **Hence proved.**

7. LHS $= \sin^2\theta \tan\theta + \cos^2\theta \cot\theta + 2\sin\theta\cos\theta$

TIP

Do simplify only one side at a time.

$= \sin^2\theta \dfrac{\sin\theta}{\cos\theta} + \cos^2\theta \cdot \dfrac{\cos\theta}{\sin\theta} + 2\sin\theta\cos\theta$

$= \dfrac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{(\sin^2\theta + \cos^2\theta)^2}{\cos\theta\sin\theta}$ $(\because a^2 + b^2 + 2ab = (a+b)^2)$

$= \dfrac{1^2}{\cos\theta\sin\theta} = \dfrac{1}{\cos\theta\sin\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

RHS $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{(\sin^2\theta + \cos^2\theta)^2}{\cos\theta\sin\theta} = \dfrac{1}{\cos\theta\sin\theta}$

$(\because \sin^2\theta + \cos^2\theta = 1)$

So, LHS = RHS **Hence proved.**

8. LHS $= \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{\dfrac{\sin A}{\cos A} - \dfrac{\cos A}{\cos A} + \dfrac{1}{\cos A}}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\cos A} - \dfrac{1}{\cos A}}$

[dividing numerator and denominator by cos A]

$= \dfrac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$

$= \dfrac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1}$

$= \dfrac{(\tan A + \sec A - 1)}{(\tan A - \sec A) + (\sec^2 A - \tan^2 A)}$

$(\because \sec^2 A - \tan^2 A = 1)$

$= \dfrac{(\tan A + \sec A - 1)}{(\tan A - \sec A) + (\sec A - \tan A)(\sec A + \tan A)}$

$= \dfrac{(\tan A + \sec A - 1)}{(\tan A - \sec A)(1 - \sec A - \tan A)}$

$= \dfrac{(\tan A + \sec A - 1)}{-(\tan A - \sec A)(\tan A + \sec A - 1)}$

$= \dfrac{1}{\sec A - \tan A} =$ RHS **Hence proved.**

9. Given, $\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ ...(1)

and $\dfrac{x}{a}\sin\theta - \dfrac{y}{b}\cos\theta = 1$ ...(2)

TIP

Adequate practice of identities is necessary to avoid errors in simplification.

Squaring eqs. (1) and (2) and then adding, we get

$\left(\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta\right)^2 + \left(\dfrac{x}{a}\sin\theta - \dfrac{y}{b}\cos\theta\right)^2 = 1+1$

$\Rightarrow \dfrac{x^2}{a^2}\cos^2\theta + \dfrac{y^2}{b^2}\sin^2\theta + \dfrac{2xy}{ab}\sin\theta\cos\theta$

$+ \dfrac{x^2}{a^2}\sin^2\theta + \dfrac{y^2}{b^2}\cos^2\theta - \dfrac{2xy}{ab}\sin\theta\cos\theta = 2$

$\Rightarrow \dfrac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \dfrac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$

$\Rightarrow \dfrac{x^2}{a^2} \times 1 + \dfrac{y^2}{b^2} \times 1 = 2 \Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$(\because \sin^2\theta + \cos^2\theta = 1)$

Hence proved.

COMMON ERROR

Sometimes students don't apply these formulae:
$(a+b)^2 = a^2 + b^2 + 2ab$ and $(a-b)^2 = a^2 + b^2 - 2ab$.
They directly simplify equation which leads to incorrect result.

10. LHS $= \sin A(1 + \tan A) + \cos A(1 + \cot A)$

TIP

Follow step-by-step simplification to avoid errors.

$= \sin A\left(1 + \dfrac{\sin A}{\cos A}\right) + \cos A\left(1 + \dfrac{\cos A}{\sin A}\right)$

$= \sin A + \dfrac{\sin^2 A}{\cos A} + \cos A + \dfrac{\cos^2 A}{\sin A}$

$= (\sin A + \cos A) + \dfrac{\sin^2 A}{\cos A} + \dfrac{\cos^2 A}{\sin A}$

$= (\sin A + \cos A) + \left(\dfrac{\sin^3 A + \cos^3 A}{\sin A \cdot \cos A}\right)$

$$= (\sin A + \cos A) + \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cdot \cos A)}{\sin A \cdot \cos A}$$

$[\because (a^3 + b^3) = (a + b)(a^2 - ab + b^2)]$

$$= (\sin A + \cos A)\left\{1 + \frac{(1 - \sin A \cdot \cos A)}{\sin A \cdot \cos A}\right\}$$

$(\because \sin^2 A + \cos^2 A = 1)$

$$= \frac{\sin A + \cos A}{\sin A \cdot \cos A}(\sin A . \cos A + 1 - \sin A \cdot \cos A)$$

$$= \frac{\sin A + \cos A}{\sin A \cdot \cos A} \times 1 = \frac{\sin A}{\sin A \cdot \cos A} + \frac{\cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A} = \sec A + \operatorname{cosec} A = \text{RHS}$$

Hence proved.

COMMON ERROR

Sometimes students don't apply this formula: $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$. They directly simplify equation which leads to incorrect result.

11. Given, $\sin\theta + \cos\theta = m$ ...(1)

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = m^2$$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = m^2$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow 1 + 2\sin\theta\cos\theta = m^2$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sin\theta\cos\theta = \frac{m^2 - 1}{2}$ ...(2)

and $\sec\theta + \operatorname{cosec}\theta = n$

$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = n$

$\Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = n$

From eqs. (1) and (2), we get

$$\frac{m}{\frac{(m^2 - 1)}{2}} = n \Rightarrow \frac{2m}{m^2 - 1} = n$$

$\Rightarrow n(m^2 - 1) = 2m$ **Hence proved.**

12. Given, $\sec\theta + \tan\theta = m$ ...(1)

$\because \sec^2\theta - \tan^2\theta = 1$

$\therefore (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m}$ [from eq. (1)] ... (2)

Adding eqs. (1) and (2), we get

$$2\sec\theta = m + \frac{1}{m} \Rightarrow \sec\theta = \frac{m^2 + 1}{2m}$$

Subtracting eq. (2) from eq. (1), we get

$$2\tan\theta = m - \frac{1}{m} \Rightarrow \tan\theta = \frac{m^2 - 1}{2m}$$

We have, $\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{m^2 - 1}{2m} \times \frac{2m}{m^2 + 1}$

$$\left[\because \frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \sin\theta\right]$$

So, $\sin\theta = \frac{m^2 - 1}{m^2 + 1}$ **Hence proved.**

COMMON ERROR

Students should be remember above method and don't solve the equation $\sec\theta + \tan\theta = m$ like as

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = m \Rightarrow \frac{1 + \sin\theta}{\cos\theta} = m$$

This type of solution make more difficult steps and consuming time in examination.

13. Given, $\tan A = n\tan B \Rightarrow n = \frac{\tan A}{\tan B}$

TIP

Adequate practice of identities is necessary to avoid errors in simplification.

and $\sin A = m\sin B \Rightarrow m = \frac{\sin A}{\sin B}$

Now, RHS $= \frac{m^2 - 1}{n^2 - 1} = \frac{\frac{\sin^2 A}{\sin^2 B} - 1}{\frac{\tan^2 A}{\tan^2 B} - 1}$

$$= \frac{\frac{\sin^2 A}{\sin^2 B} - 1}{\frac{\sin^2 A/\cos^2 A}{\sin^2 B/\cos^2 B} - 1}$$

$$= \frac{\frac{\sin^2 A - \sin^2 B}{\sin^2 B}}{\frac{\sin^2 A \cdot \cos^2 B}{\sin^2 B \cdot \cos^2 A} - 1} = \frac{\frac{\sin^2 A - \sin^2 B}{\sin^2 B}}{\frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\sin^2 B}}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B}$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{(\sin^2 A - \sin^2 B)}$$

$= \cos^2 A = \text{LHS}$ **Hence proved.**

COMMON ERROR

Sometimes students forget the identity used and hence commit error.

Chapter Test

Multiple Choice Questions

Q 1. If cosec $\theta = \sqrt{10}$, then sec θ is equal to:

a. $\frac{\sqrt{10}}{3}$ b. $\sqrt{10}$

c. $\frac{3}{\sqrt{10}}$ d. $\frac{2}{\sqrt{10}}$

Q 2. The value of $(\sin 30° + \cos 30°) - (\sin 60° + \cos 60°)$ is:

a. -1 b. 0

c. 1 d. 2

Assertion and Reason Type Questions

Directions (Q.Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): In ΔABC, right angled at B, if $\sin A = \frac{8}{17}$, then $\cos A = \frac{15}{17}$ and $\tan A = \frac{8}{15}$.

Reason (R): For acute angle θ, $\cos\theta = \frac{\text{Hypotenuse}}{\text{Base}}$, and $\tan\theta = \frac{\text{Base}}{\text{Perpendicular}}$.

Q 4. Assertion (A): If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^2 A = 2$.

Reason (R): $1 - \sin^2 A = \cos^2 A$, for any value of A.

Fill in the Blanks

Q 5. The maximum value of $\sin\theta$ is ....

Q 6. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is ....

True/False

Q 7. $\sin^2\theta + \cos^2\theta = 1$ is a trigonometry identity.

Q 8. The value of $\cos\theta$ decreases from 1 to 0 when θ increases from 0° to 90°.

Case Study Based Question

Q 9. An electrician wanted to repair a street lamp which is at a height of 15 feet. He places his ladder such that its foot is 8 feet from the foot of the lamp post as shown in the figure below:

Based on the above information, solve the following questions:

(i) Find the value of cos R.

(ii) Find the value of cosec P.

(iii) Find the value of $\frac{\sin R - \cos P}{\sin R + \cos P}$.

Or

Find the value of $\tan R + \frac{3}{\sec P} - 1$.

Very Short Answer Type Questions

Q 10. Is it possible that $\sin\theta = \frac{15}{11}$?

Q 11. Simplify $9\sec^2\theta - 9\tan^2\theta$.

Short Answer Type-I Questions

Q 12. If $\sec\theta + \tan\theta = 9$, then find $(\sec\theta - \tan\theta)$.

Q 13. Prove that $(1 + \sin A)(\sec A - \tan A) = \cos A$.

Short Answer Type-II Questions

Q 14. If $\tan\theta + \sin\theta = m$ and $(\tan\theta - \sin\theta) = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$.

Q 15. Prove that

$$\cot^2 A\left(\frac{\sec A - 1}{\sin A + 1}\right) + \sec^2 A\left(\frac{\sin A - 1}{\sec A + 1}\right) = 0.$$

Long Answer Type Question

Q 16. Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.